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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

STRAINS, STRESSES, AND SHEAR IN ENGINEERING PROBLEMS

By Silas H. Woodard,[1] M. Am. Soc. C. E.

Synopsis

Formulas for strains and stresses on planes oblique to direction of principal stresses are developed in this paper. The thesis is offered that stresses are limited to tensile and compressive stresses; that shearing stress and shearing strength are misnomers; that shear is resisted by simple diagonal tensile and compressive stresses; that ruptures or failures occur when material is overstressed in tension; and that all failures in the field of engineering structures (except where plastic flow is classed as a failure) are tensile failures.

Fundamental Problems of Stress in Terms of Strain

Let Fig. 1 be the side elevation of a prism $A_1B_1C_1D_1$ of length H, and 1 unit by 1 unit in cross section, acted upon by force, p, parallel with the axis—that is, with the dimension H. During the application of the force, p, the prism changes from $A_1B_1C_1D_1$ to $A_2B_2C_2D_2$ or vice versa depending on whether p is compression or tension; the length of $\frac{H}{2}$ changes to the length $\left(\frac{H}{2} - h\right)$; the dimensions normal to the axis change to the length $(1 - 2\,b)$; the length of $a = \overline{OB_1}$ changes to the length $(d = \overline{OB_2})$.

Let δ = strain in direction of $p = 2\,\frac{h}{H}$; δ_2 = (strain normal to direction of p) $= 2\,\frac{b}{1} = 2\,b$; δ_3 = (strain in direction of $\overline{OB_1}$)

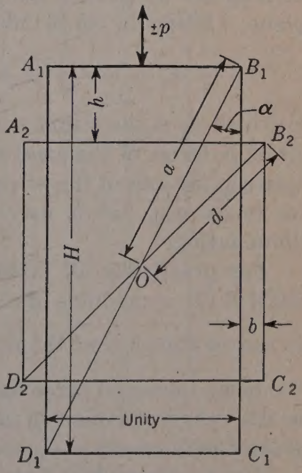

Fig. 1

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by July 1, 1943.

[1] Cons. Engr., New York, N. Y.

$= \frac{a - d}{a} = 1 - \frac{d}{a}$; δ_n = (strain normal to direction of $\overline{\mathrm{OB}}_1$); $\mu = \frac{\delta_2}{\delta}$; and α = the angle between the direction of p and $\overline{\mathrm{OB}}_1$. Furthermore,

$$\sin \alpha = \frac{1}{2\,a} \qquad \text{(1a)}$$

$$\cot \alpha = \frac{H}{1} = H \qquad \text{(1b)}$$

$$a = \frac{1}{2 \sin \alpha} \qquad \text{(1c)}$$

$$h = \frac{\delta \cot \alpha}{2} \qquad \text{(1d)}$$

and

$$d = \sqrt{\left(\frac{H}{2} - h\right)^2 + (\tfrac{1}{2} + b)^2} \qquad \text{(2a)}$$

Substituting values of H, h, and b, in Eq. 2*a*,

$$d = \tfrac{1}{2} \sqrt{(1 - \delta)^2 \cot^2 \alpha + (1 + \mu\,\delta)^2} \qquad \text{(2b)}$$

and

$$\delta_3 = 1 - \frac{d}{a} = 1 - \sin \alpha \sqrt{(1 - \delta)^2 \cot^2 \alpha + (1 + \mu\,\delta)^2} \qquad \text{(3)}$$

Simplifying Eq. 3,

$$\delta_3 = 1 - \sqrt{(1 + \mu\,\delta)^2 + [(1 - \delta)^2 - (1 + \mu\,\delta)^2] \cos^2 \alpha} \qquad \text{(4a)}$$

Usually the designer is interested in the stresses and strains normal to a given plane. From Eq. 4*a* it follows that

$$\delta_n = 1 - \sqrt{(1 + \mu\,\delta)^2 + [(1 - \delta)^2 - (1 + \mu\,\delta)^2] \sin^2 \alpha} \qquad \text{(4b)}$$

in which δ_n is the strain normal to any plane cutting a prism subject to axial stress in terms of the axial strain, Poisson's ratio μ and the sine of the angle α between the axis of the prism and the cutting plane. Eq. 4*b* is rather awkward for ready use, but it may be plotted into curves that are both useful and illuminating.

For practically all materials of construction the elastic limit is less than 0.004 E (E = modulus of elasticity) and working stresses are less than 0.001 E. It can be shown that the ratio, $\frac{\delta_n}{\delta}$, if computed by Eq. 4*b*, will be the same to the second decimal place for all values of δ less than 0.005. The uncertainties in the values of Poisson's ratio and the modulus of elasticity would vitiate any greater refinement.

The curves of Fig. 2 show, for various values of Poisson's ratio and for all angles, the strain normal to a plane that cuts a prism which is stressed axially, in terms of the strain in the direction of the axis as computed by Eq. 4*b*. The curves are for the following values of Poisson's ratio:

μ	Material
0.1	An approximate value for neat cement mortar
0.15	An approximate value for 1 : 3 mortar
0.2	An approximate value for ordinary concrete
0.25	An approximate value for stone
0.275	Value frequently used for steel
0.3	Another reported determination for special steel
0.33	Value generally used for copper
0.40	Value reported for lead

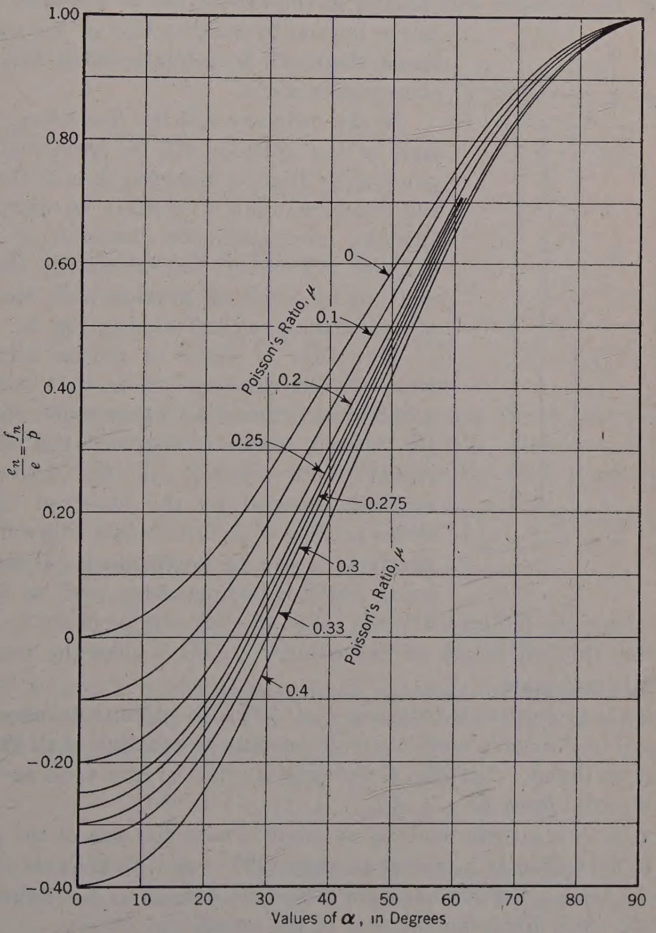

FIG. 2.—STRESS NORMAL TO A PLANE THAT CUTS A PRISM WHICH IS STRESSED AXIALLY

Interpolation may be made for other values of Poisson's ratio by simple proportion. If stress is proportional to strain, the curves of Fig. 2 can be used to read not only the strains normal to any plane cutting a prism in terms of axial strain of the prism but also the stresses, in terms of the principal axial stress, p.

In Fig. 2, the ordinates are marked to read both $\frac{\delta_n}{\delta}$ and $\frac{f_n}{p}$, in which f_n is the stress normal to the inclined plane. Of course, if p is tension, the ordinates above the x-axis will indicate tension and those below, compression. If p is compression, the ordinates above will indicate compression and those below will indicate tension.

Consider the case of a test specimen of a concrete cylinder or prism being subjected to axial pressure in a compression testing machine. By theory the diameter of the cylinder will expand in the proportion of Poisson's ratio and, acting normal to the direction of the compressive stress, there will be a tensile stress of μ times the compressive stress.

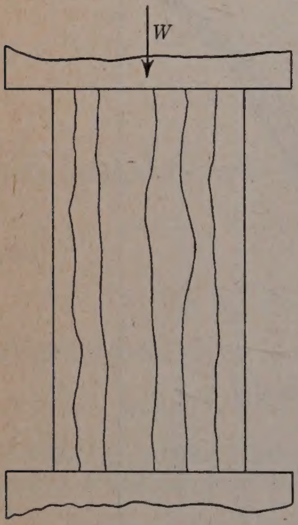

FIG. 3

In the ordinary test the diameters of the two ends of the cylinder will be prevented from expanding by friction between it and the heads of the machine; but if friction be eliminated by effective lubrication, the diameter of the entire cylinder will expand and ultimately the material will fail by cracking parallel with the direction of compression, as indicated in Fig. 3.

Hundreds of tests[2] of mortar and concrete have proved this and show that such failures occur when μ times the compression stress equals the known tensile strength of the concrete; or stated more exactly, f_t, the known tensile strength, divided by the observed compressive stress at time of failure equals Poisson's ratio as nearly as it can be determined. If the material being tested is homogeneous, such as 28-day neat cement mortar, the failure will often cause the cylinder or prism to break into small needles the full length of the cylinder. This is generally true of amorphous brittle materials.

If stress is proportional to strain and if strains produce transverse strains proportional to Poisson's ratio, a simple formula or equation of all the curves of Fig. 2 may be found. Let Fig. 4 represent a prism of unit cross sections acted upon by an axial force p.

Section A–A is a plane making an angle α with the axis of the prism; and plane B–B, normal to A–A, makes an angle $(90° - \alpha)$ with the axis of the prism. Resolve the force p into two components—one normal to A–A, the other normal to B–B—and divide by the area of the planes and obtain:

The normal force on unit area of plane A–A,

$$p_a = p \sin^2 \alpha \ldots\ldots\ldots\ldots\ldots\ldots (5a)$$

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[2] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1092.

and the normal force on unit area of plane B–B,

$$p_b = p \cos^2 \alpha \ldots\ldots\ldots\ldots (5b)$$

Then by hypothesis the force p_a produces a strain normal to A–A, of

$$\delta_a = \frac{p_a}{E} = \frac{p \sin^2 \alpha}{E} \ldots\ldots\ldots\ldots (6a)$$

but, force p_b produces a strain of opposite sign normal to A–A, of

$$\delta_b = -\frac{\mu\, p_b}{E} = -\frac{\mu\, p \cos^2 \alpha}{E} \ldots\ldots (6b)$$

The net strain normal to plane A–A is $\delta_a + \delta_b = \delta_n$. Then

$$\delta_n = \frac{p}{E} (\sin^2 \alpha - \mu \cos^2 \alpha)$$

$$= \frac{p}{E} [(1 - \mu) \sin^2 \alpha - \mu] \quad (7)$$

and

$$f_n = E\, \delta_n = p\, [(1 + \mu) \sin^2 \alpha - \mu] \ldots (8a)$$

Another form for Eq. 8*a* is,

$$f_n = p\, [1 - (1 + \mu) \cos^2 \alpha] \ldots\ldots (8b)$$

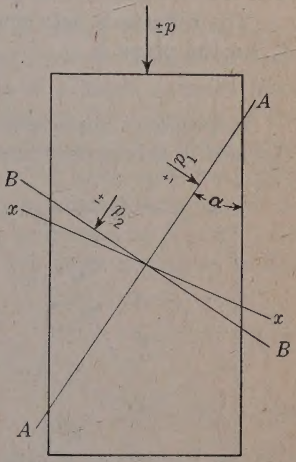

FIG. 4

In Eqs. 8*a* and 8*b*, p is the axial stress in a prism and f_n is the stress normal to a plane making an angle α with that axis. These formulas plot curves identical with those of Fig. 2. However, they can be derived directly from Eq. 4*b*. For example, write Eq. 4*b* in the form

$$\delta_n - 1 = -\sqrt{(1 + \mu\, \delta)^2 + [(1 - \delta)^2 - (1 + \mu\, \delta)^2] \sin^2 \alpha} \ldots\ldots (9a)$$

Squaring both sides of Eq. 9*a* and expanding the binomials:

$$\delta^2_n - 2\, \delta_n + 1 = 1 + 2\, \mu\, \delta + \mu^2\, \delta^2$$
$$+ (1 - 2\delta + \delta^2 - 1 - 2\, \mu\, \delta - \mu^2\, \delta^2) \sin^2 \alpha \ldots\ldots\ldots (9b)$$

But, δ^2_n, $\mu^2\, \delta^2$, and δ^2 are extremely small fractions, of the order of less than one millionth. Also they tend to cancel each other by opposite signs. If these can be dropped, Eq. 9*b* becomes

$$\frac{\delta_n}{\delta} = 1 - (1 + \mu) \cos^2 \alpha = \frac{f_n}{p} \ldots\ldots\ldots\ldots (10)$$

which is the same as Eq. 8*b*. The evidence indicates that Eq. 10 is applicable for all materials of construction up to the strains at which rupture occurs.

STRESS AND INTENSITY OF FORCE

It is common to conceive, and sometimes to define, stress as the normal component of the force known to be transferred across a unit area of cross section of a material. Referring to Fig. 4, the normal components of the force,

p, upon unit area of the planes A–A and B–B are

$$p_a = p \sin^2 \alpha \qquad (11a)$$

$$p_b = p \cos^2 \alpha \qquad (11b)$$

$$p_a + p_b = p \qquad (11c)$$

The normal stress f_a on plane A–A is given by Eq. 8*b*.

The difference between the intensity of normal force, p, and normal stress, f_a, on the plane is

$$p_a - f_a = \mu\, p \cos^2 \alpha \qquad (12)$$

It becomes important to distinguish between stress and force intensity. As used in this paper, stress f is defined as the magnitude of internal force normal to unit area which is accompanied by, and proportional to, strain in the same direction; or conversely, if the distance between two particles in homogeneous matter be changed, the proportional change is a measure of the stress on a unit area normal to the straight line between the particles. Force intensity p is defined as the sum of the normal components of all forces acting on unit area.

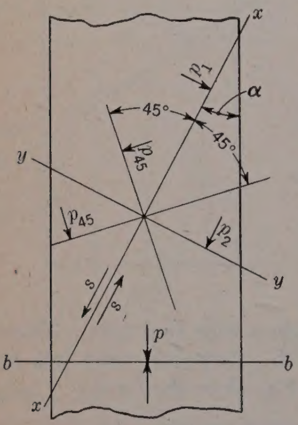

Fig. 5

Stress f may be greater than, equal to, or less than, force intensity p. This study indicates that f may be greater or less than p by a percentage not greater than Poisson's ratio, the variation being determined by the angles between the direction of principal stresses and the plane on which the stress or force intensity is acting.

Force intensity, although not a measure of stress, is a necessary and useful unit in problems of mechanics, and will be used in all appropriate equations of applied mechanics. Stress and force intensity are numerically the same when the stress is a single, maximum, principal stress, as, for example, p in Figs. 1 and 5. Furthermore, stress and force intensity are always numerically the same in any and every direction in a material for which Poisson's ratio is zero—but there is probably no such material.

The writer hereby enlists with the partisans of the Maximum Stress and Maximum Strain Theories of failure as opposed to the Maximum Shear Theory, the Maximum Distortion Energy Theory, and the Maximum Strain-Energy Theory. Although they are inseparable, the Maximum Strain Theory and Maximum Stress Theory are sometimes referred to as two independent theories.

Shear and Stress

It is common to use the terms shearing stress and shearing strength, but two logical deductions in the following study are that, (*a*) in materials strained by exterior forces including shearing forces, only two kinds of stresses are set up—compressive stresses and tensile stresses—and (*b*) shear is accompanied by and resisted by internal forces of tension and compression. In other words, there is no such thing as shearing stress or shearing strength as distinguished from compressive and tensile stresses and compressive and tensile strength.

However, the same deductions develop a rigid relation between shear and tensile and compressive stress so that shear and shearing intensities may be used to compute stresses.

Consider the simple case illustrated by Fig. 5 in which (x–x) is a plane making an angle α with the direction of a principal stress p (tension or compression). This stress is assumed to be the result of a single exterior force acting in the same direction. On plane (b–b) normal to the direction of the exterior force there is no shear and the stress is a maximum and is the same as the force intensity. Suppose the cross section (b–b) to have unit area, then the exterior force will also be p. The force intensity p_a normal to (x–x) will be as expressed by Eq. 5*a*, and the stress f_a will be as expressed by Eq. 8*b*. On plane (y–y) normal to (x–x), the force intensity p_b will be as expressed by Eq. 5*b*, and the stress f will be

$$f_b = p\,[1 - (1 + \mu)\sin^2] \ldots\ldots\ldots\ldots (13a)$$

The unit shear on either (x–x) or (y–y) will be

$$s = p \sin\alpha \cos\alpha \ldots\ldots\ldots\ldots (13b)$$

The force intensity normal to one of the planes making an angle of 45° with (x–x) and (y–y) will be (see Eq. 5*a*):

$$p_{45} = p \sin^2(\alpha + 45) = p\,(\tfrac{1}{2} + \sin\alpha\cos\alpha) \ldots\ldots\ldots (14a)$$

and normal to the other it will be

$$p_{-45} = p \sin^2(\alpha - 45) = p\,(\tfrac{1}{2} + \sin\alpha\cos\alpha) \ldots\ldots\ldots (14b)$$

Eqs. 14*a* and 14*b* represent two forces acting diagonally across the shear planes. The net effect of the components of p_{45} and p_{-45} parallel to the shear planes is their algebraic difference. These components on unit area of the shear planes are found by multiplying Eqs. 14*a* and 14*b* by $\sin^2 45°$, with the result that the net effect is found to be, $p \sin\alpha \cos\alpha$, which is the value of unit shear as given by Eq. 13*b*. In other words, the force intensities p_{45} and p_{-45} of Eqs. 14*a* and 14*b* are real forces and constitute a sufficient answer to the question of how shear is resisted without calling upon an imaginary and inexplicable thing called shearing stress.

If the shear per unit area on any pair of shear planes and the force intensity on each of the planes is known, then the direction and intensity of the maximum and minimum stresses can be computed (keeping in mind that the maximum and minimum stresses usually mean the maximum tensile and compressive stress or the minimum tensile and compressive stress and that the engineer is principally interested in finding the maximum stress to compare it with the known breaking stress). Let p_a = the force intensity on one shear plane; p_b = the force intensity on the other shear plane; s = the shear per unit area of the shear planes; μ = Poisson's ratio for the stressed material; and β and

$(90° - \beta)$ = the angles between the shear planes and any other plane such as (x–x), Fig. 4.

The stress $f_{(x-x)}$ on plane (x–x), then, is equal to

$$f_{(x-x)} = p_a\,[1 - (1 + \mu)\sin^2\beta] + p_b\,[1 - (1 + \mu)\cos^2\beta] \pm 2\,s\,(1 + \mu)\sin\beta\cos\beta \qquad (15)$$

Eq. 15 follows if it be assumed that there are four principal stresses acting separately in the material under consideration, p_a and p_b acting normal to the shear planes and $+\,p_c$ and $-\,p_c$ (numerically equal to s) acting at 45° to the shear planes. Then by determining the stress on the plane (x–x) making an angle β and $(\beta - 90°)$ with the shear-plane equation, by use of the preceding equations, Eq. 15 is obtained. To find the value of β which corresponds with the maximum value of f, place the first differential of f with respect to $\beta = 0$.

$$\frac{\delta f}{\delta \beta} = -\,2\,(1 + \mu)\,p_a \sin\beta\cos\beta + 2\,(1 + \mu)\,p_b \sin\beta\cos\beta - 2\,(1 + \mu)\,s\sin^2\beta + 2\,(1 + \mu)\,s\cos^2\beta = 0 \qquad (16)$$

from which

$$\sin^2\beta = \tfrac{1}{2} \pm \tfrac{1}{2}\sqrt{\frac{(p_b - p_a)^2}{(p_b - p_a)^2 + 4\,s^2}} \qquad (17)$$

Having found the values of β from Eq. 17, they may be substituted in Eq. 15 and the maximum and minimum stress, f, determined. As stress is numerically the same as force intensity p, if Poisson's ratio, μ, is zero, then

$$p = p_a\cos^2\beta + p_b\sin^2\beta \pm 2\,s\sin\beta\cos\beta \qquad (18)$$

The two values of p (maximum and minimum force intensities) obtained from Eq. 18 are frequently called the principal stresses, but, according to the definition of stress in this paper, they are not equal to stress except in special cases; however, they are important and usable values. In the special case where $s^2 = p_a \times p_b$ (see Fig. 4), the maximum values of p and f are the same but not the minimum values.

When using Eqs. 15 and 17 it is usually more convenient to express p_a and p_b in terms of s. If $p_a = k_1\,s$ and $p_b = k_2\,s$,

$$\sin^2\beta = \tfrac{1}{2} \pm \tfrac{1}{2}\sqrt{\frac{(k_2 - k_1)^2}{(k_2 - k_1)^2 + 4}} \qquad (19)$$

Consider some special examples of pairs of shear planes subject to unit shears equal to s and normal force intensities of p_a and p_b, respectively.

Example 1.—The case of pure shear on a given plane, such as at the neutral axis of a simple beam or on the cross section of a shaft subjected to torsion only. Here $p_a = 0$ and $p_b = 0$, and Eq. 17 becomes $\sin^2\beta = \frac{1}{2}$, or $\beta = 45°$; $f = \pm\,(1 + \mu)\,s$, and $p = \pm\,s$. That is, there will be a unit stress of $+\,(1 + \mu)\,s$ on one of the planes making an angle of 45° with the shear plane and a unit stress of $-\,(1 + \mu)\,s$ on the other; and on the same planes there will be a force intensity, $p = \pm\,s$.

Example 2.—In the case of a column or simple tension member under uniform axial stress equal to p; then $p_a = k\,s$; $p_b = \frac{s}{k}$; or $p_a\,p_b = s^2$. This will be true for every shear-plane section, and k will be equal to tan α, in which α is the angle between the shear plane and the axis of the member. Then, if β is the angle between a shear plane and the plane of maximum stress:

$$\sin^2\beta = \tfrac{1}{2} \pm \tfrac{1}{2}\sqrt{\frac{k^2 - 1}{k^2 + 1}} = \cos^2\alpha \quad\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Eq. 20 is sometimes useful, and it furnishes confirmation of Eqs. 8*b*, 15, and 18.

Example 3.—The case of a combination of shear with one simple stress, such as a shaft under torsional unit shear s, and an axial tension or compression of, say, $p_b = k\,s$, then

$$\sin^2\beta = \tfrac{1}{2} \pm \tfrac{1}{2}\sqrt{\frac{k^2}{k^2 + 4}} \quad\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

TABLE 1.—MAXIMUM AND MINIMUM STRESSES (IN POUNDS PER SQUARE INCH) COMPUTED BY EQS. 15 AND 17

(Proof Tensile Strength 23,400 Lb per Sq In.)

Test No.	Applied shear	Applied tension	Sin² β	β	COMPUTED STRESS	
					Tensile	Compressive
3SA3	10,300	15,700	0.8031	63°–39′	22,500	10,900
3SA24	10,300	14,900	0.7927	62°–53′	21,910	11,110
3SA15	11,060	13,200	0.7510	60°– 0′	21,520	12,680
3SA18	11,060	13,800	0.7643	60°–57′	21,980	12,500
3SA21	11,060	14,300	0.7715	61°–26′	22,250	12,300
3SA4	12,860	11,700	0.7087	67°–26′	22,710	15,550
3SA20	12,860	14,100	0.7399	59°–21′	24,230	15,010
3SA10	12,720	10,300	0.6894	56°– 6′	21,750	15,700
3SA17	12,720	9,950	0.6823	55°–41′	21,500	15,750
3SA7	13,100	0	0.5000	45°– 0′	17,500	17,500

Table 1 shows the maximum and minimum stresses by Eqs. 15 and 17 for nine tests of combined torsion and tension on samples of aluminum, reported by Joseph Marin, Assoc. M. Am. Soc. C. E., and R. L. Stanley[3] in 1940.

The value of Poisson's ratio μ for the material tested is assumed to be 0.33. The proof tensile strength, 23,400 lb per sq in., was the mean of three tests which varied by 4.4%.

It will be seen that the computed maximum tensile stresses agree with the proof tensile strength as closely as the test results in each group agree with each other. Unfortunately, there were no compression tests to determine the proof compressive strength. The single pure shear test (torsion only) yielded at a unit shear of 13,100 lb per sq in., for which the stress computed by Eqs. 15 and 17 would be 17,500 lb per sq in. compression on one diagonal and 17,500 lb per sq in. tension on the other. It may also be noted that the stress-strain curves in the determination of the "proof strength" indicate the yield point to be about 17,500 lb per sq in. Possibly the compressive proof strength would be approximately 17,500 lb per sq in.

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[3] "Failure of Aluminum Subjected to Combined Stresses," by Joseph Marin and R. L. Stanley, *Supplement, The Welding Journal*, February, 1940, Vol. 19, pp. 74s–80s.

CONCLUSIONS

It is obvious that—

(*a*) Poisson's ratio must be used in the more accurate computations of stresses; and

(*b*) Shear intensity is not a stress but is always accompanied and resisted by diagonal tensile or compressive stresses or both.

Other conclusions may be deduced from the formulas developed herein, therefore, such as,

(*c*) If shear exists on a plane section at a given point, then the maximum or principal stress at that point is not normal to the plane. This fact is frequently overlooked;

(*d*) If the principal stress is known at any point, the stresses in all other directions may be found;

(*e*) Conversely, if the shearing and force intensities on a pair of shear planes at any point be known, the directions and magnitude of the maximum or principal stresses may be found;

(*f*) The stresses in all other directions may be computed either from the principal stresses or direct from the shear and force intensities.

Both from the formulas herein and from practical observation, it is concluded that ruptures of material under compression are tensile failures; also, that tests to determine compressive strength as usually made, especially those on mortars and concrete, give incorrect results and indicate too high strength because of the resisting effect of friction on the ends. If this is true, the testing methods should be revised to eliminate friction which can be done easily, or a correction factor should be determined to apply to the results of tests as now made.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

DETERMINATION OF KUTTER'S *n* FOR SEWERS PARTLY FILLED

By C. Frank Johnson,[1] Assoc. M. Am. Soc. C. E.

Synopsis

Kutter's formula is widely used in computations involving the hydraulics of sewers. The value of the coefficient of roughness, *n*, in this formula can be determined for sewers by only one method—making actual measurements of velocities (or discharges) and depths of flow in the sewers. The results of measurements made in pipes and conduits carrying clean water cannot be used for sewers without modification; however, they do afford a valuable guide. Unfortunately, records of determinations of the value of *n* for sewers are all too scarce.

This paper presents the results of certain determinations, based on field measurements, of the value of Kutter's *n* for sewers partly filled. The technique followed in making the field measurements is described, and a comparison is made between the various methods used for measuring the velocities of flow in the sewers.

Introduction

During extensive gagings of sewage flows in Louisville, Ky., in 1939–1940, field determinations were made of the velocities of flow and corresponding discharges in four sewers at various depths of flow ranging in different sewers from 0.03 D to about 0.4 D. From the results of these determinations Kutter's *n* was computed for each test run, and an average value determined for each sewer. Several conclusions based on the results of these determinations are given at the close of this paper. The one of most general interest, probably, indicates that the values of *n* for low flows in sewers are higher than the values for flows of greater depth. The fact that higher values of *n* obtain for pipes not substantially filled has been recognized for some time,[2,3] however, there is a scarcity of experimental data showing the actual values of *n* for sewers having low flows.

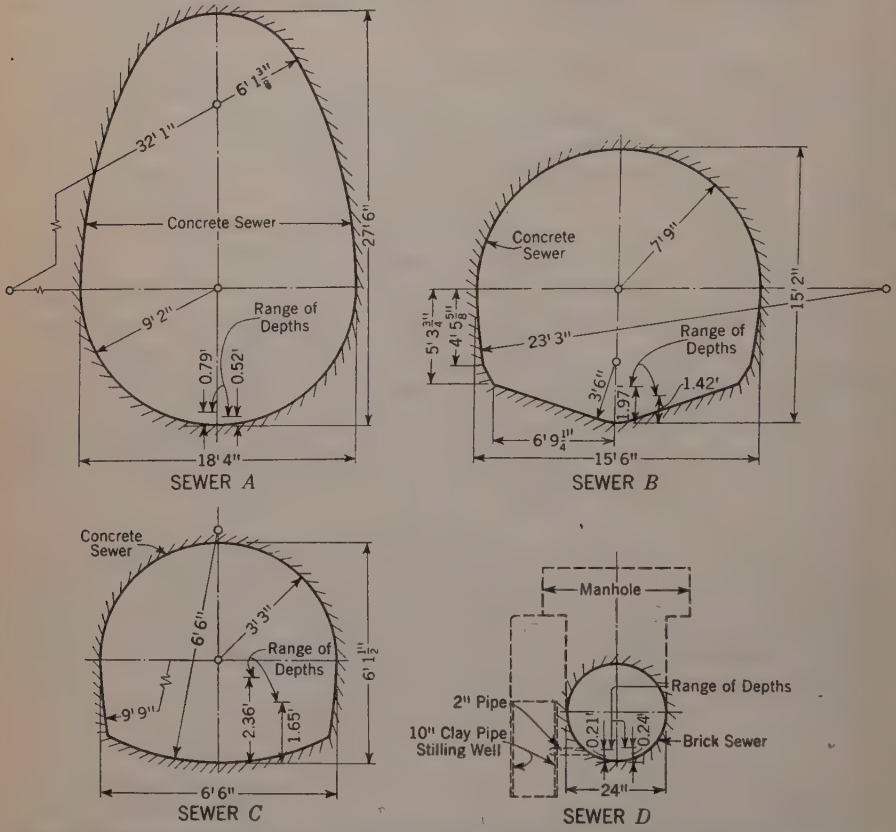
Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **July 1, 1943.**

[1] Chf. Engr., Commrs. of Sewerage of Louisville, Louisville, Ky.

[2] "On the Proper Value of Kutter's *n* for Sewer Computations," by Charles W. Sherman, *Engineering News-Record*, Vol. 88, No. 23, June 8, 1922, p. 948.

[3] "Hydraulic Formulas Incorrect for Partly Filled Pipes," by Charles W. Sherman, *ibid*, Vol. 103, No. 7, August 15, 1929, p. 253.

The sewers in which the measurements were made are shown in Fig. 1. These sewers were considered to be representative of average city sewers with respect to cleanliness; they had never been cleaned by city forces, so far as was known, nor was there any indication that they particularly needed cleaning.



SEWER *A* SEWER *B*

SEWER *C* SEWER *D*

Sewer	Interior Dimensions (Ft)		Shape	Age, in years	Nominal slope	Length, in feet[a]
	Horizontal	Vertical				
A	18.33	27.50	Inverted egg	7	0.00087	1,550
B	15.50	15.17	Horseshoe	30	0.0005	890
C[b]	6.50	6.13	Horseshoe	31	0.0005	1,060
D[c]	2.00	2.00	Circular	66	0.0324	260

[a] Approximate length of test reach. [b] An intercepting sewer. The others are combined sewers. [c] A brick sewer. The others are concrete sewers.

FIG. 1.—CHARACTERISTICS OF SEWERS IN WHICH MEASUREMENTS WERE MADE

The measurements of velocities and discharges described herein were made on various dates during dry weather, when the sewage presumably contained no storm water; nor was there any ground water entering the sewers within the test reaches, according to available data. Although Sewer A (Fig. 1) was partly below the ground-water level, lead-strip water stops incorporated in the transverse construction joints prevented the entrance of ground water. The other sewers were above the ground-water level. It was assumed that the flows were not increased by ground water entering within the test reaches.

GENERAL TECHNIQUE

Velocities were measured by the dye-velocity (or color) and the salt-velocity methods. Each measurement was made by noting the time of travel of the dye or salt solution from the point of application to a downstream point of observation. The solution, after thorough mixing, was poured into the flowing sewage as rapidly as possible, and the average time of the application noted either on a stop watch or by the use of the second hand on an ordinary watch. Observations at the downstream point were made by the use of the same watch or another watch synchronized with the one used at the upstream point.

Dye-Velocity Method.—When using dye, the time of its appearance at the downstream station was noted visually, as was also the approximate time of the most intense color and the time when the dye could last be detected. The exact time of the appearance of the dye was easily determined, but the exact times of the most intense color and the last appearance of the dye were difficult to determine, because of the gradual changing or fading of the color at these times. Also, the natural discoloration of the sewage and the necessity for using artificial light in the sewers hampered these visual determinations somewhat.

The dye-velocity method has previously been utilized extensively for measuring velocities of both sewage and clean water and has been shown[4] to give results averaging within about 3% of those obtained by the use of carefully constructed weirs. The time of travel of the dye is generally considered to be the interval between the instant of introduction of the solution and the average time of the first and last appearance of the color at the downstream station.[4,5]

Salt-Velocity Method.—When using salt, the time of passage of the salt at the downstream station was determined either by the chemical method—that is, by titrating small samples of sewage taken at definite times—or by the electrical-resistance method, using electrodes immersed in the flowing sewage.

For the chemical or titration method, samples were dipped from the flowing sewage near the center of the sewer at a point just below the surface. Except on two runs, the samples were taken at 10-sec intervals, starting 1 or 2 min before the computed time when a pilot dye test indicated that the first salt should reach the downstream station and continuing until 1 or 2 min after the like computed time of the last appearance of the salt. The samples were titrated for chloride content, using standard methods with silver nitrate as a reagent and potassium chromate as an indicator. A graph of the chloride content of the sewage was then plotted, using time as abscissas and chloride content as ordinates. The center of gravity of the area included between the line of normal chloride content extended and the line showing increased chloride content was found by cutting out the area from a piece of cardboard and suspending it from first one point and then another, marking the center of gravity at the intersection of the directions of the suspending cord. Then, on the graph the position of the vertical through this center of gravity, called the centroid vertical, was plotted, the location of the centroid vertical being considered to represent the mean time of passage of the salt particles.[6,7]

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[4] "The Flow of Water in Concrete Pipe," by Fred C. Scobey, *Bulletin No. 852*, U.S.D.A., 1920, p. 19.

[5] "Handbook of Hydraulics," by Horace W. King, 2d Ed., New York, N. Y., 1929, pp. 372 and 373.

On the later runs, when using salt, the electrical-resistance method was used and found to be much easier, faster, and more satisfactory than titration of samples. Copper electrodes of 24-gage metal, 2 in. square, were mounted on a lead-weighted wooden strip at a spacing of about 1 in. apart. The electrodes were connected to one or more small dry cells, with a variable resistance in the line. Meters reading to 300 milliamperes and to 8 volts were used. The electrodes were suspended in the sewage, all other equipment and the observers being above ground. Readings of the milliammeter were taken at 2-sec or 5-sec intervals and were plotted in the same manner as the results of the titrations, previously described. The centroid vertical of the area of increased current resulting from the passage of the salt charge was taken to indicate the time of passage of the salt.[5]

The salt-velocity method has previously been used extensively for measuring the velocity of clean water in pipes under pressure and has been shown to give results, under such conditions, averaging only 0.32% in excess of a weir.[6] This method has been utilized for testing large pumps, after its accuracy had been verified by volumetric check runs.[8] However, the salt-velocity method does not appear to have been applied to a great extent for measuring velocities in sewers partly filled.

Depth-of-Flow Measurements.—For each test the depth of flow of the sewage was ascertained by measuring from a previously established reference mark or grade nail, set above the water at a known short distance above the sewer invert, to the water surface. This measurement was subtracted from the height of the reference mark above the invert. The height above the invert of each reference mark was determined in advance by measuring the flow depths of the sewage at several points in the vicinity, by averaging these, and by adding to the average depth the distance from the reference mark to the water surface at the time.

In Sewers A, C, and D (Fig. 1), the depths for the tests were measured at the downstream end of the reach, check measurements having indicated that the depths so determined represented the mean depth in the reach. In Sewer B, however, it was necessary to measure the depths at the upstream and downstream ends of the reach, as these depths varied as much as 0.07 ft; in this case, the average areas and average wetted perimeters were used.

Determination of Slopes.—The slope of the water surface in Sewers A, C, and D was considered to be the same as that of the sewer, because the check measurements previously mentioned had indicated that this was substantially correct. After they were checked by levels run in the field, the nominal slopes of these three sewers were used. However, in the case of Sewer B, the measured slope of the water surface (corrected to the slope of the energy gradient on two runs where the correction was as much as 0.01 ft) was utilized.

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[6] "The Salt Velocity Method of Water Measurement," by C. M. Allen and E. A. Taylor, *Transactions*, A.S.M.E., Vol. 45, 1923, p. 285.

[7] "Study of Sewage Settling Tank Design," by C. H. Capen, Jr., *Engineering News-Record*, Vol. 99, No. 21, November 24, 1927, p. 833.

[8] "Building Big Pumps Better Earns A Bonus," *ibid.*, Vol. 125, No. 11, September 12, 1940, p. 85.

Determination of n.—Based on the velocities determined in the sewers, computations were made of the value of C in the Chezy formula:

$$V = C \sqrt{R S} \qquad (1)$$

in which V = velocity, C = coefficient, R = hydraulic radius, and S = slope of the energy gradient (or water surface). The results of these computations furnished data for the determination of Kutter's n by the use of tables in standard handbooks.

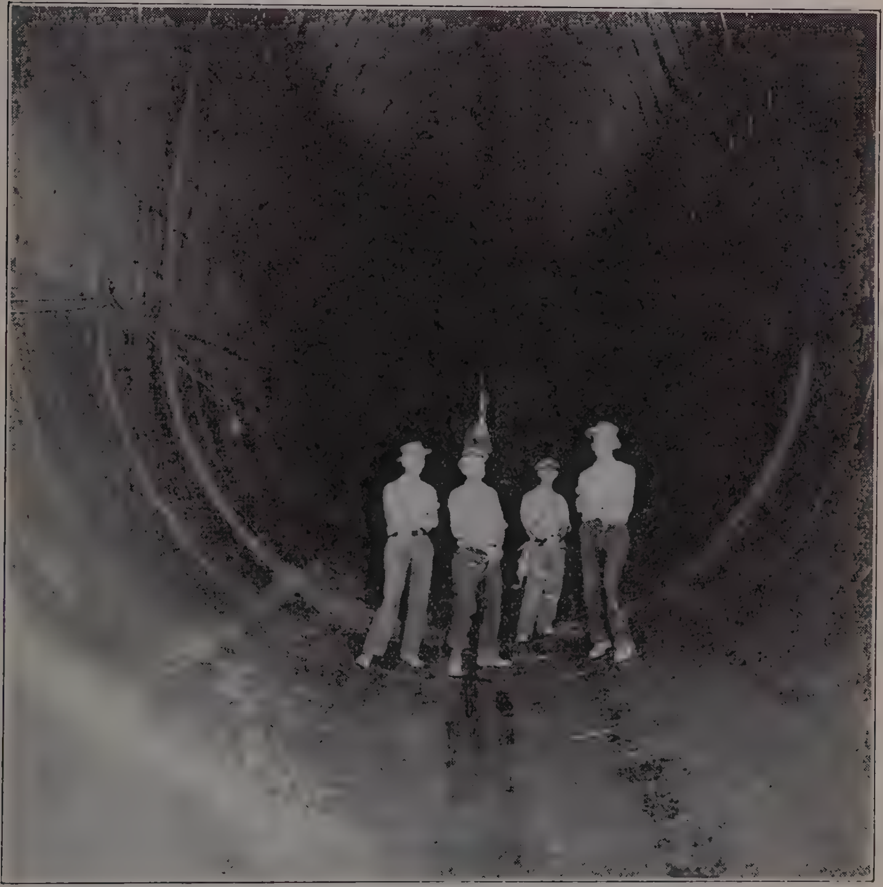

FIG. 2.—SEWER A, BEFORE BEING PUT INTO SERVICE

SEWER A

Description and Condition of Sewer.—Built in 1932, Sewer A is part of the Southwestern Outfall. Within the test reach the sewer is straight except for one curve which has a radius of 100 ft and which is 14 ft long. There are no "live" inlets within the reach. The invert is unlined concrete except for three stretches each 13.5 ft long, which are bricklined; and the concrete of the invert (up to the springing line) was originally poured against steel forms.

At the time of the tests the condition of this sewer was excellent in every way (see Fig. 2). The concrete of the invert was very smooth, being considerably better than average in this respect, and there was no deposit on the invert. The evenness of the grade was shown by a profile of the invert, and by measured depths of flow, which had a maximum variation of only 0.10 ft within the test reach at any one time. This was not considered excessive for such a large sewer.

Parshall Measuring Flume.—The dye-velocity and salt-velocity methods were checked in this sewer by use of a Parshall measuring flume[9] which was in place at the upstream end of the test reach. The check was made to ascertain just what stage or time of the passage of the dye or salt should be used for determining the mean velocity for these particular tests and to check the accuracy of the methods used.

The flume was so constructed that "free-flow" conditions were secured, necessitating the measurement of only one head, H_a, to determine the discharge. For each test, the head was measured immediately after pouring the solution into the sewage.

General Results.—The results of the titration of the sewage samples taken in Sewer A are shown graphically in Fig. 3. The principal data and the results of these tests, as well as of those made by use of the dye, are given in Table 1(a). The velocities determined by use of the flume, and the corresponding values of Kutter's n, are shown in Table 1(b). A comparison of these velocities with

TABLE 1.—COMPARISON OF CHEMICAL METHOD WITH
CORRESPONDING VALUES

(a) DETERMINATIONS BY USING DYE AND SALT

Test No.[a]	Chemical Used		Water (gal)[d]	Depth[e] d (ft)	Ratio[f] $\frac{d}{D}$	Section area[g] A (sq ft)	Wet perimeter[g] P (ft)	Hydraulic radius[g] R (ft)	Velocity[g] (Ft Per Sec) Based on:		
	Kind[b]	Lb[c]							Peak content V_p	Centroid vertical[h] V_c	Time of appearance[g] V_m
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
A-1	Dye	1.0	6	0.76	0.0415	3.73	7.54	0.495	1.92	....	1.80
A-2	Dye	4.5	15	0.78	0.0426	3.88	7.67	0.506	1.91	....	1.84
A-3	Salt	15.0	10	0.52	0.0284	2.12	6.25	0.339	1.50	1.44	1.36
A-4	Salt	30.0	15	0.77	0.0421	3.81	7.60	0.501	1.94	1.91	1.86
A-5	Salt	55.0	18	0.79	0.0431	3.96	7.73	0.512	1.99	1.95	1.87
Average		....	....	....	....	....	....	....	....	....	....

[a] The length of reach was 1,553 ft in each test except No. A-3, in which it was 1,592 ft. The slope of the water sodium chloride. [c] Approximate weight. [d] Dissolving water required. [e] Mean depth of flow, d. [f] Col. appearance for dye tests, and on centroid vertical of area of increased chloride content for salt tests. [j] From "The equals Q/A, or Col. 18 ÷ Col. 7. [l] Based on hydraulic elements in Cols. 5 to 9, with S = 0.00087. [m] $\frac{V_p - V_f}{V_f}$

those determined by use of dye and salt is given in Table 1(*c*). (In Fig. 3, Test No. A-4: B = beginning of increased chloride content; P = peak of salt content; E = end of increased chloride content; *M* = mean time of salt appearance; and the line BE is the line of normal chloride content.)

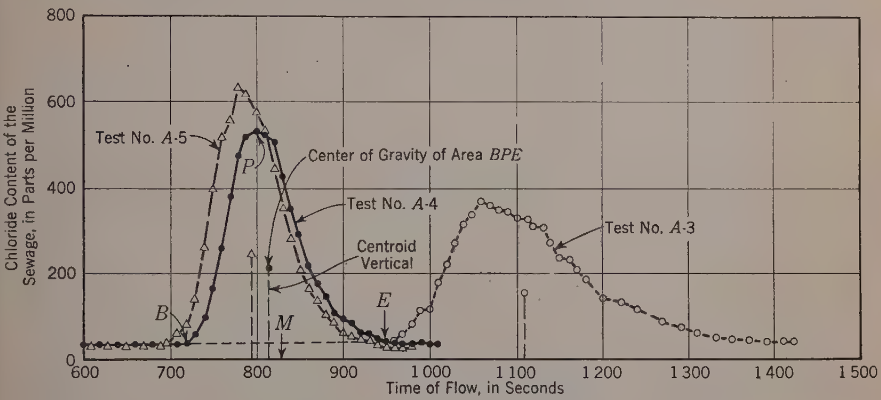

FIG. 3.—TITRATION OF SEWAGE SAMPLES FROM SEWER A

It will be observed that in Table 1(*c*) no comparison is shown for Test No. A-3. The results of this test are viewed with some question for two reasons: (1) The head on the flume (0.28 ft) was very near the recommended minimum[9]

MEASURING FLUME, FOR DETERMINING VELOCITIES, AND THE
OF *n*, FOR SEWER A

				(b) FIVE-FOOT PARSHALL FLUME				(c) COMPARISON (%)			
n, in Kutter's Formula, Based on:			Q (cu ft per sec)[i]	Observed head H_a (ft)	Q (cu ft per sec)[j]	V_f (ft per sec)[g,k]	Kutter's[l] *n*	Peak content[m]	Centroid vertical[n]	Mean time[o]	Test No.[a]
Peak content	Centroid vertical[h]	Time of appearance[g]									
(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(1)
0.0138	....	0.0146	6.71	0.48	6.24	1.67	0.0154	15.0	....	7.8	A-1
0.0142	....	0.0146	7.14	0.52	7.09	1.83	0.0147	4.4	....	0.5	A-2
0.0135	0.0140	0.0146	3.05	0.28	2.65	1.25	0.0156	....	....	....	A-3
0.0139	0.0141	0.0144	7.28	0.52	7.09	1.86	0.0143	4.3	2.7	0.0	A-4
0.0138	0.0140	0.0144	7.72	0.53	7.30	1.84	0.0146	8.2	6.0	1.6	A-5
0.0138	0.0140	0.0145	....	....	....	....	0.0149	8.0	4.4	2.5	Average

surface was 0.00087. [b] "Dye" denotes soluble Prussian blue dye (sodium ferri-ferrocyanide). "Salt" denotes
5 ÷ 18.33 ft. [g] Mean values. [h] Area of increased chloride content. [i] Discharge based on the mean time of
Parshall Measuring Flume," by Ralph L. Parshall, Assoc. M. Am. Soc. C. E.[9] [k] Velocity of flow in the sewer,
$= \frac{\text{Col. 10} - \text{Col. 19}}{\text{Col. 19}}$. [n] $\frac{V_e - V_f}{V_f} = \frac{\text{Col. 11} - \text{Col. 19}}{\text{Col. 19}}$. [o] $\frac{V_m - V_f}{V_f} = \frac{\text{Col. 12} - \text{Col. 19}}{\text{Col. 19}}$.

for a 5-ft flume; and (2) it appeared that some of the salt solution might have been retained unduly long at the point of application. This point was located at the downstream end of the flume, where some of the dye on a preceding test (which was rejected) had been retained for a short time, by eddies in the sewage flow. Furthermore, the chloride-content graph for Test No. A-3 (Fig. 3) is very flat after passing the peak, apparently from the same cause. This difficulty was obviated on the other tests by pouring the solution into the sewage at a point approximately 40 ft downstream from the flume.

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[9] "The Parshall Measuring Flume," by Ralph L. Parshall, *Bulletin No. 423*, Colorado State Coll., Colorado Experiment Station, Fort Collins, Colo., 1936.

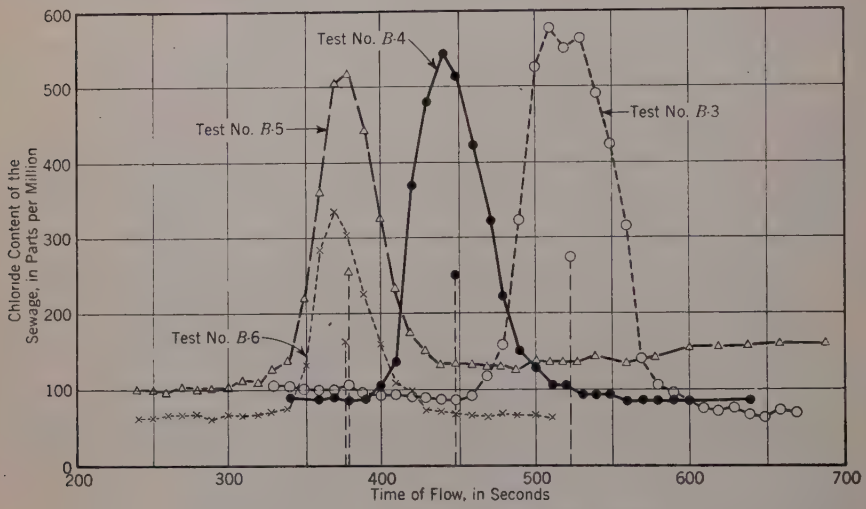

FIG. 4.—TITRATION OF SEWAGE SAMPLES FROM SEWER B

The comparison shown in Table 1(*c*) indicates a satisfactorily close check for the purposes of the gagings being made, the use of the centroid vertical on Tests Nos. A-4 and A-5 indicating velocities averaging 4.4% in excess of those obtained by use of the flume. According to Mr. Parshall,[9] the accuracy of discharge measurements made with the Parshall measuring flume is within 2% to 5%, under normal operating conditions. A closer check between the flume measurements and the dye and salt measurements might have been secured during the tests by taking the sewage samples for titration from some point in the cross section other than near the center of the sewer, say, at or near the quarter point. As the samples were taken near the center of the sewer, however, the computed velocities may be as much as 2% to 5% in excess of the respective true mean velocities, because there is some chance that only the fast water was thus timed. Such difference has been found when using the salt-velocity method in this way for measuring velocities in pipes entirely filled.[6] For these reasons, the velocities determined by use of the centroid vertical of the salt tests, made as described herein, seemed slightly above the true average velocities, and the resulting values of *n* were therefore slightly low. It was thought that these values of *n* were as much as, say, 2% to 5% low. (This possibility should be kept in mind when studying the results of the tests.) The use of a slightly low value of *n* was not objectionable from the standpoint of the purpose of the sewage gagings being made—to obtain data upon which to base the design of intercepting sewers and a sewage-treatment plant—as the computed discharges would then be slightly, but not significantly, on the high or safe side.

In addition to the other considerations, the results of the salt-velocity tests, rather than those of the dye-velocity tests, were used for the determinations of the mean velocities and the values of *n*, because the salt method affords more tangible data and gives more definite and satisfactory results than the dye method. For this reason, and as a result of the comparisons made and discussed in the preceding paragraph, these determinations were based on the centroid vertical of the salt tests.

The dye tests were necessary in some instances as pilot tests, however, particularly when using titration of samples, and were of value in all instances as check observations. When the color method is used exclusively, the mean time of the dye appearance would probably indicate the mean velocity most satisfactorily.

Use of the centroid vertical for the salt tests and the mean time of the dye appearance for the color tests is in accord with the recorded practice of others.

Value of n.—Based on the centroid vertical, the average value of Kutter's *n* for Sewer A, at the extremely low depths of flow encountered, is 0.0140. The value of *n* for this sewer when substantially filled is probably 0.0130 or lower.

SEWER B

Description and Condition of Sewer.—Sewer B is part of the Southern Outfall, built in 1909. Within the test reach the sewer is straight, with one 15-in. inlet. However, no water seemed to be discharged by this inlet during the tests. The entire inside surface of the sewer is unlined concrete and, according to a report of the commission supervising its construction, "steel forms were used [on the side-walls] * * *, the invert having a troweled finish equivalent to that ordinarily given sidewalks."

At the time of the tests the condition of this sewer was good and the concrete surface fairly smooth, but not as smooth as the concrete of Sewer A. An inspection of the entire stretch, made about two years prior to the time when the velocity measurements were made, revealed that "most of the invert is free from deposit and is slippery"; however, a deposit about ½ in. deep was reported at the center of the sewer at three points within the stretch. At the time of the tests, conditions appeared to be substantially the same as when the inspection was made.

General Results.—The results of the titration of the sewage samples taken in Sewer B are shown graphically in Fig. 4, and in Table 2(*a*) are given the principal data and the results of all the tests conducted in this sewer.

The values of *n* based on the individual tests in this sewer are not so consistent as might be desired. This may be the result of a considerable deposit on the invert of the sewer at a point approximately 900 ft downstream from the test reach. The deposit acted as a low dam, causing the previously mentioned variations in the depth of flow within the test reach.

Value of n.—Based on the centroid vertical the average value of Kutter's *n* for Sewer B at the depths encountered is 0.0144. The value of *n* for this sewer when substantially filled is probably about 0.0130.

SEWER C

Description and Condition of Sewer.—Built in 1908, this sewer is part of the Beargrass Intercepter. Within the test reach the sewer is straight for only about one third of its length. However, the curves are of such long radii (more than 500 ft) that the curvature seems to have had little or no effect on the value of n. There is one 8-in. inlet and one 20-in. lateral sewer within the stretch. It is not known whether the 8-in. inlet was discharging any water during the tests, but the 20-in. lateral sewer was discharging a volume equal probably to about 10% of the total volume flowing in Sewer C. However, this lateral is situated so near the upper end of the test reach (71 ft downstream) that the resulting error due to the addition of this flow within the reach should be slight. The entire surface of the sewer is unlined concrete. The concrete of the side-walls was originally poured against wooden forms and the invert was probably given a troweled finish similar to that given Sewer B.

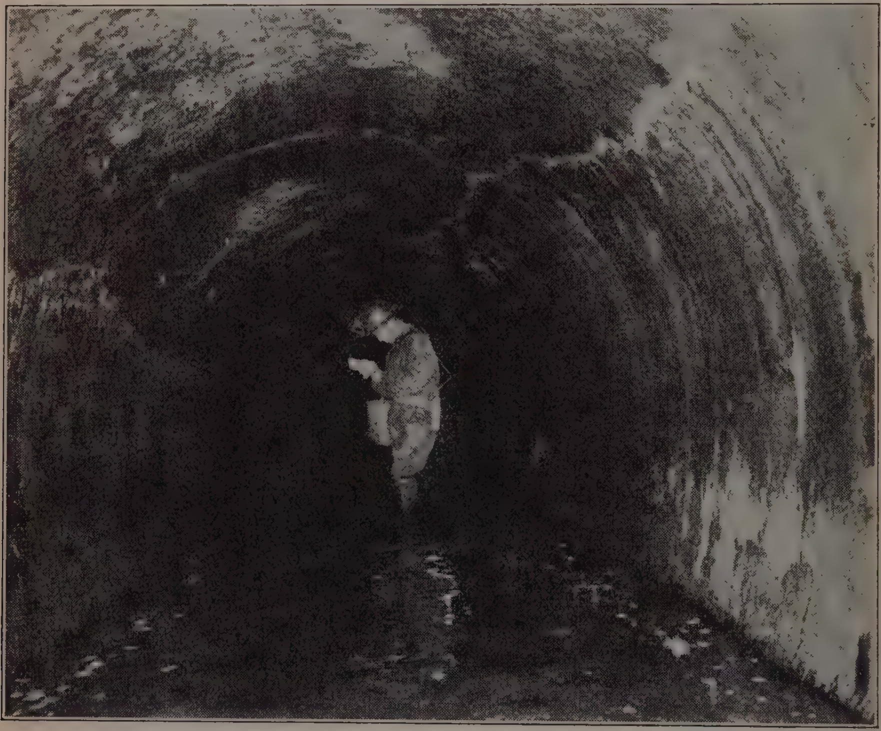

FIG. 5.—A VIEW IN SEWER C, WITH A PORTION OF THE FLOW DIVERTED TEMPORARILY

At the time of the tests the condition of this sewer was good, and the concrete surface fairly smooth (see Fig. 5)—about as smooth as the concrete of Sewer B. An inspection, made with most of the flow temporarily diverted,

TABLE 2.—DETERMINATIONS OF VELOCITIES AND CORRESPONDING VALUES OF n IN SEWERS B, C, AND D

Test No.	CHEMICAL USED		Water (gal)[c]	Length of reach (ft)	Slope[d] S	Depth[d,e] d (ft)	Ratio[f] $\frac{d}{D}$	Section areas[e] A (sq ft)	Wet perimeter[e] P (ft)	Hydraulic radius[e] R (ft)	VELOCITY[g] V (FT PER SEC) BASED ON:		n, IN KUTTER'S FORMULA, BASED ON:		Q (cu ft per sec)[h]
	Kind[a]	Lb[b]									V_c	V_m	Centroid vertical	Time of appearance[e]	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
							(a) SEWER B (15 FT 6 IN. BY 15 FT 2 IN.—CONCRETE)								
B-1	Blue	0.8	5	892	0.00049	1.86	0.123	12.19	12.64	0.964	....	2.10	....	0.0152	25.6
B-2	Blue	2.0	15	892	0.00051	1.97	0.130	13.62	13.36	1.019	....	2.25	....	0.0150	30.6
B-3	Salt[i]	40	14	892	0.00049	1.42	0.094	7.51	9.94	0.756	1.70	1.67	0.0155	0.0159	12.8
B-4	Salt[i]	45	14	892	0.00049	1.58	0.104	9.07	10.92	0.831	1.99	1.88	0.0144	0.0152	18.1
B-5	Salt[i]	55	16	892	0.00052	1.82	0.120	11.74	12.40	0.947	2.34	2.26	0.0139	0.0144	27.5
B-6	Salt[i]	60	18	892	0.00052	1.84	0.121	11.98	12.52	0.956	2.36	2.29	0.0139	0.0143	28.3
							(b) SEWER C (6 FT 6 IN. BY 6 FT 1½ IN.—CONCRETE)								
C-1	Fluor	0.13	2	1,057	0.0005	2.34	0.382	13.06	9.50	1.375	....	2.77	....	0.0148	36.2
C-2	Fluor	0.13	2	1,057	0.0005	1.68	0.274	8.82	8.18	1.078	....	2.44	....	0.0143	21.5
C-3	Salt[i]	40	17	1,057	0.0005	2.36	0.385	13.18	9.54	1.382	2.89	2.78	0.0144	0.0148	38.1
C-4	Salt[i]	40	17	1,057	0.0005	1.70	0.277	8.95	8.22	1.089	2.44	2.46	0.0145	0.0144	21.8
C-5	Salt	30	15	1,060	0.0005	1.65	0.269	8.63	8.12	1.063	2.66	2.60	0.0131	0.0134	23.0
C-6	Salt	30	15	1,060	0.0005	1.67	0.273	8.76	8.16	1.074	2.61	2.60	0.0135	0.0135	22.9
							(c) SEWER D (24 IN.—BRICK)								
D-1	Blue	0.13	1	261	0.0324	0.23	0.115	0.201	1.384	0.145	....	3.33	....	0.0176	0.67
D-2	Blue	0.13	1	261	0.0324	0.23	0.115	0.201	1.384	0.145	....	3.44	....	0.0172	0.69
D-3	Blue	0.13	1	261	0.0324	0.24	0.120	0.214	1.415	0.151	....	3.33	....	0.0180	0.71
D-4	Salt	15	5	262	0.0324	0.24	0.120	0.214	1.415	0.151	3.91	3.63	0.0160	0.0169	0.84
D-5	Salt	10	4	262	0.0324	0.235	0.118	0.207	1.399	0.148	3.68	3.44	0.0166	0.0174	0.76
D-6	Salt	10	4	262	0.0324	0.235	0.118	0.207	1.399	0.148	3.61	3.40	0.0168	0.0175	0.75
D-7	Salt	10	4	262	0.0324	0.21	0.105	0.176	1.320	0.133	3.38	3.19	0.0166	0.0173	0.59
D-8	Dichr	2.5	5	262	0.0324	0.21	0.105	0.176	1.320	0.133	3.56	3.36	0.0159	0.0166	0.63

[a] "Blue" denotes soluble Prussian blue dye (sodium ferri-ferrocyanide); "salt" denotes sodium chloride; "fluor" denotes fluorescein dye; and "dichr" denotes potassium dichromate. [b] Approximate weight. [c] Dissolving water required. [d] Slope of water surface, S, and mean depth of flow, d. [e] Mean values. [f] Col. 7 ÷ D (in which D = vertical diameter of the sewer). [g] V_c = mean value, centroid vertical; and V_m = mean time of appearance. [h] Based on the mean time of dye appearance for the dye tests, and on the centroid vertical for the salt and dichromate tests. [i] Titration of samples was used for these tests. The electrical-resistance method was used for other salt (and dichromate) tests.

two years prior to the time of making the velocity measurements, revealed that this part of the sewer was in good condition, with the exception of a 50-ft stretch where the arch was severely cracked. The gravel was loose along these cracks, and there were also quite a few transverse hair cracks and a few longitudinal hair cracks for short stretches (as in Fig. 5). The defective conditions would probably have little or no effect on the value of n under the conditions of the tests made. During this same inspection two short stretches of deposit, both in the vicinity of the 20-in. lateral, were discovered. One stretch extended 50 ft, averaging about 2 in. deep at the center of the sewer, and the other stretch extended 20 ft, averaging about 1 in. deep. Conditions at the time of the tests were substantially the same as when the inspection was made.

General Results.—Two salt-velocity tests were made in Sewer C by the titration method and two by the electrical method. The results of the electrical determinations are shown graphically in Fig. 6. The principal data and the results of all the tests made in this sewer are given in Table 2(*b*). As in Fig. 3, Points B, P, and E for Test No. C-5 denote, respectively: The beginning of increased chloride content; the peak of the salt content; and the end of the increased chloride content. The line of normal chloride content is marked BE, and M denotes the mean time of salt appearance.

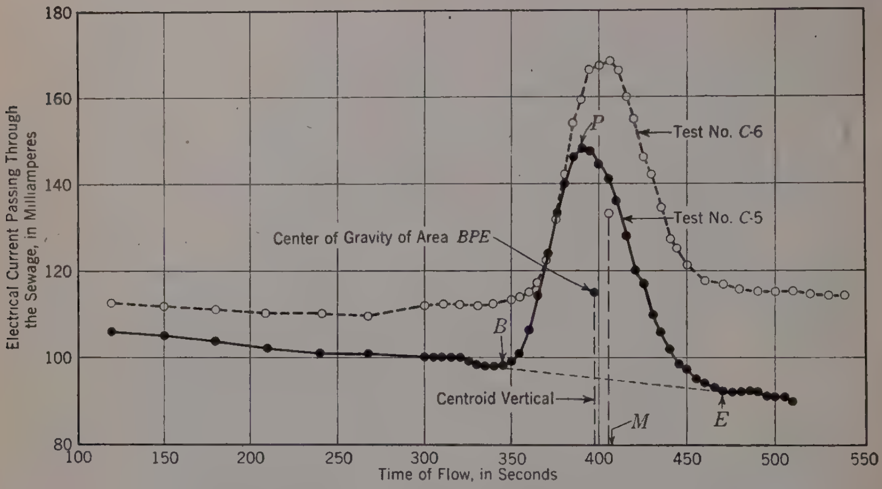

Fig. 6.—Electrical Determinations of Increased Chloride Content in Sewer C

Table 2(*b*) shows that there is a comparatively wide difference between the results of the first series of tests in this sewer, using dye and titration of sewage samples, and the results of the second series, using electrical methods with salt. No particular reason is known for this difference. The two series were made on different dates when there may have been a variation in the lateral sewer discharge or possibly in some other factor not apparent. The accuracy of the

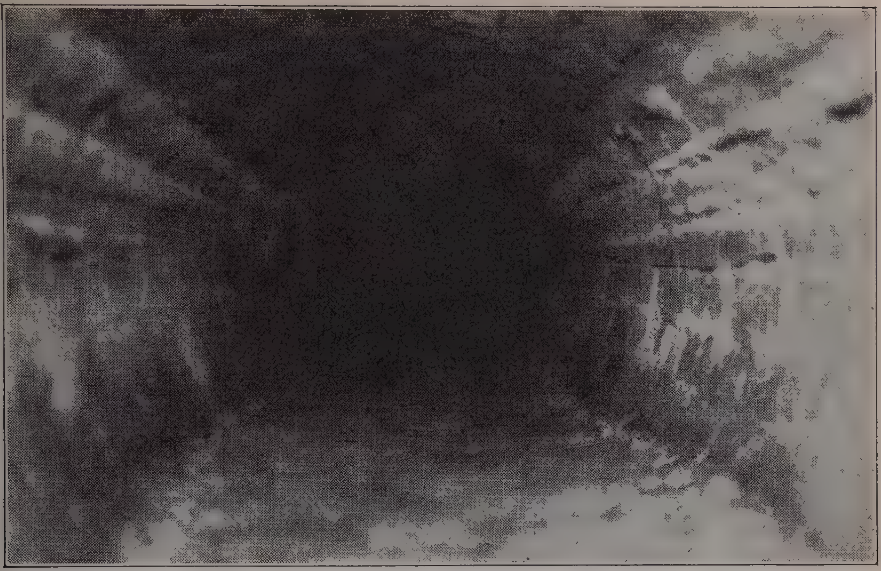

FIG. 7.—A VIEW IN SEWER D, WITH FLOW CONDITIONS SIMILAR TO THOSE DURING VELOCITY MEASUREMENTS

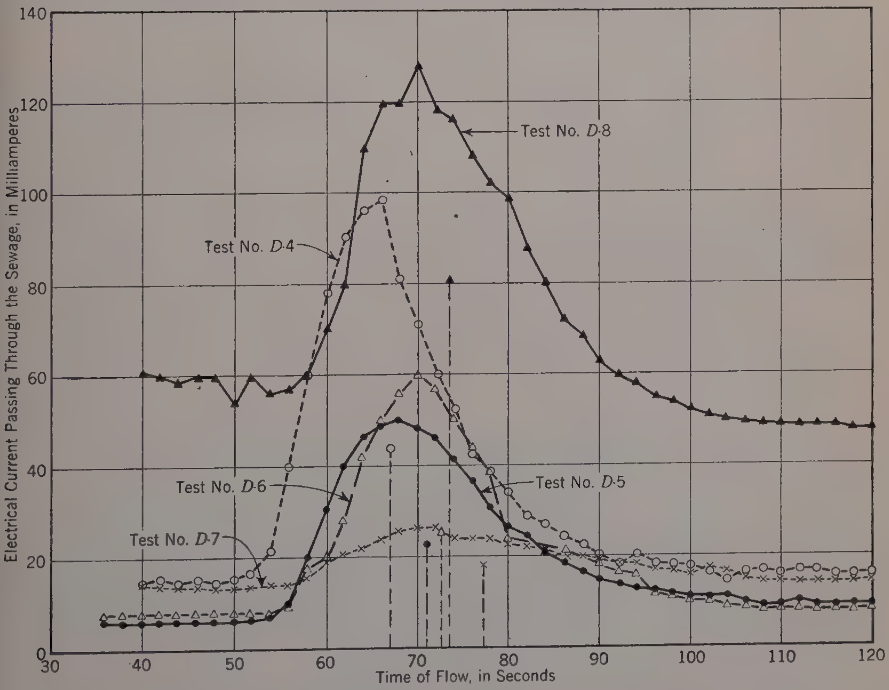

FIG. 8.—ELECTRICAL DETERMINATIONS OF INCREASED CHLORIDE CONTENT IN SEWER D

methods used during the second series certainly exceeded that of the first series; the crew was also considerably more experienced when making the second series. For these reasons the values of *n* determined from the later tests (Nos. C-5 and C-6) appeared the more reliable.

Value of n.—Based on the centroid vertical the average value of Kutter's *n* for Sewer C, at the depths encountered, is 0.0139. However, the value accepted for use in the gaging work is 0.0135, giving more weight to the values determined from Tests Nos. C-5 and C-6—namely, 0.0131 and 0.0135. The value of *n* for this sewer when substantially filled is probably about 0.0130.

SEWER D

Description and Condition of Sewer.—This is the old brick sewer in Third Street, north of Main Street, constructed in 1873. Within the test reach the sewer is straight and on a uniform grade. There was little or no sewage being discharged into the sewer within this stretch while the tests were being made. At the time of the tests the condition of this sewer was good, considering its age (see Fig. 7). In general, the joints were well filled, and there were few pronounced irregularities in the brick surface. The walls were fairly straight—in fact, it was easy to see from manhole to manhole.

General Results.—Electrical methods were used exclusively for the salt tests in this sewer, since, in view of the extremely short time of flow (varying from 67 to 77 sec), it would have been impossible to take sewage samples sufficiently fast to obtain accurate results. As an experiment potassium dichromate was used, successfully, on the last test. The results of these tests are shown graphically in Fig. 8, and the principal data and the results of all the tests made in this sewer are given in Table 2(*c*). The results of the individual tests made in this sewer are fairly consistent in spite of the extremely short time of flow.

TABLE 3.—Summary of Aver

No.	Kutter's *n*, based on:	Sewer A		
		Dye	Salt	All
1	Peak, dye or salt content	0.0140	0.0137	0.0138
2	Centroid vertical	....	0.0140[a]	0.0140
3	Mean time, dye or salt appearance	0.0146	0.0145	0.0145
4	Discharge of flume	0.0151	0.0148	0.0149
5	Value adopted for gaging work	0.0140		

Value of n.—Based on the centroid vertical the average value of Kutter's *n* for Sewer D, at the depths encountered, is 0.0164. This value was accepted for use. The value of *n* for this sewer when substantially filled is probably about 0.0150.

Summary of Average Values of n

The average values of Kutter's n for all the sewers are summarized in Table 3, which indicates that the average value based on the centroid vertical of the salt tests was accepted for the gaging work in every instance except one. The reason for the variation in this instance has been given.

Comparison of Kutter's n with Manning's n

For comparison, the values of Kutter's n based on the centroid vertical of the salt-velocity tests are shown in Table 4 with the corresponding values of n in the Manning formula.[10]

In the case of Sewers B and C, the respective values of n are practically the same. These two sewers had the largest hydraulic radii of the four. There is a small difference in the two values for Sewer A, but the greatest difference is found in the case of Sewer D, for which the hydraulic radius was the least. In the light of similar comparisons previously made,[10] this relation between the values of n in the two formulas is not abnormal.

Conclusions

1. In general, higher values of n should be used for low flows in sewers than for flows of greater depths. The values given in Table 5, based largely on the tests described herein, are suggested for use in the absence of experimental data indicating other values. The values shown in the table are fairly consistent with such other tests on sewers as have come to the writer's attention, with a few exceptions. However, the values are higher than those found by others for conduits carrying clean water, probably because of the thin coating of slime and grease deposited on the sewer walls by sewage over a period of years.

2. The reasons for higher values of n for low flows are not clear. Friction between the free surface of the water and the air above has been suggested.[3]

ge Values of Kutter's n

Sewer B			Sewer C			Sewer D			No.
Dye	Salt	All	Dye	Salt	All	Dye	Salt	All	
....	....	....	....	....	....	....	....	....	1
....	0.0144[a]	0.0144	....	0.0139	0.0139	....	0.0164[a]	0.0164	2
0.0151	0.0150	0.0150	0.0146	0.0140	0.0142	0.0176	0.0171	0.0173	3
[a] This value was selected as the acceptable value for gaging work.									4
0.0144			0.0135			0.0164			5

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[10] "Handbook of Hydraulics," by Horace W. King, 2d Ed., New York, N. Y., 1929, pp. 182, 283, and 318.

TABLE 4.—COMPARISON OF KUTTER'S *n* WITH MANNING'S *n*, BASED ON THE CENTROID VERTICAL OF THE SALT-VELOCITY TESTS

No.	Coefficient *n*	TEST NUMBERS: 3	4	5	6	7	8	Average
				(*a*) SEWER A				
1	Kutter	0.0140	0.0141	0.0140	....	....	....	0.0140
2	Manning	0.0148	0.0145	0.0144	....	....	....	0.0146
				(*b*) SEWER B				
3	Kutter	0.0155	0.0144	0.0139	0.0139	....	....	0.0144
4	Manning	0.0160	0.0146	0.0140	0.0139	....	....	0.0146
				(*c*) SEWER C				
5	Kutter	0.0144	0.0145	0.0131	0.0135	....	....	0.0139
6	Manning	0.0143	0.0144	0.0130	0.0134	....	....	0.0138
				(*d*) SEWER D				
7	Kutter	....	0.0160	0.0166	0.0168	0.0166	0.0159	0.0164
8	Manning	....	0.0194	0.0203	0.0207	0.0206	0.0195	0.0201

The coating of slime and grease on the lower sections of the sewer walls may have some influence.

3. The dye-velocity and the salt-velocity methods can be applied conveniently for measuring velocities of flow in sewers partly filled. The salt-velocity method gives more satisfactory, and probably more accurate, results than the dye-velocity method, and the use of electrical resistance for determining relative salt content is decidedly much easier and quicker than titration of samples.

TABLE 5.—SUGGESTED VALUES OF *n* FOR SEWERS HAVING NO OBSTRUCTIONS

Depth of flow (1)	*n*: Minimum (2)	Average[a] (3)	Maximum (4)
	MONOLITHIC-CONCRETE SEWERS		
0 to ⅛ full..........	0.0140	0.0150	0.0160
⅛ to ¼ full..........	0.0135	0.0145	0.0155
¼ to ½ full..........	0.0130	0.0140	0.0150
½ full to full........	0.0125	0.0130	0.0135
	OLD BRICK SEWERS		
0 to ⅛ full..........	0.0160	0.0170	0.0180

[a] Recommended values for use where inside surfaces are fairly smooth and otherwise in good condition. Lesser or greater values should be used where conditions warrant.

4. For dye-velocity tests the average time of passage of the dye is indicated best by the average time between the first and last appearance of the dye at the observation point. For salt-velocity tests the average time of passage of the salt is indicated best by the centroid vertical of the area of increased chloride content.

5. For measuring velocities in sewers partly filled, the samples for titration should not be taken, nor should the electrodes be placed, at the center of the sewer. Additional field tests are desirable to indicate the best point in the cross section for taking the samples and for placing the electrodes.

Conclusion 4 indicates that the use of the average time of passage of the dye may give results approximating those obtained by use of the centroid vertical of the salt tests. This may appear illogical. Indeed, at first glance, the results of the measurements described herein (see Table 3) do not seem to substantiate the conclusion stated. However, recalling that the values of n based on these salt tests are thought to be possibly as much as 2% to 5% low, since only the fast water is timed, the results appear more consistent in this respect. Moreover, when using dye there is little or no alternative to accepting the average time between the first and last appearance of the dye for computing the velocity. There is no other time subject to satisfactory determination, and, indeed, this time is difficult to determine satisfactorily when using the dye in a sewer. However, for the salt tests there is ample evidence supporting the use of the centroid vertical of the area of increased chloride content.

(A possible reason why the average time of the passage of the dye should agree closely with the time indicated by the centroid vertical is advanced by C. M. Allen and E. A. Taylor.[6] This reason is that the eye cannot detect the entire curve of real color but sees only the portion of the curve formed when the dye is not too diluted; that is, portions of the color curve near each end are not detected. As a rule, the average time obtained by observing the visible part of the curve agrees fairly closely with the time obtained by use of the centroid vertical of the salt tests.)

ACKNOWLEDGMENTS

The work described herein was done by the engineers of the Commissioners of Sewerage of Louisville, Ky., under the general direction of the late Woolsey M. Caye, M. Am. Soc. C. E., at that time technical engineer for the commission. Aid and advice were given by the commission's consulting engineers, Metcalf and Eddy, of Boston, Mass. The work was under the immediate direction of the writer, then senior engineer for the commission. On some of the field tests assistance was rendered by Stuart E. Coburn, chemist for Metcalf and Eddy.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

CHARACTERISTICS OF HEAVY RAINFALL IN NEW MEXICO AND ARIZONA

BY LUNA B. LEOPOLD,[1] JUN. AM. SOC. C. E.

SYNOPSIS

Seasonal differences in storm types are typical of New Mexico and Arizona. Summer storms ordinarily are of small areal extent and are characterized by high-intensity rainfall in a well-defined pattern with relation to time. Winter storms cover large areas and usually are of low intensity. In this paper, results of frequency analyses of one-day rainfalls are presented for all stations in New Mexico and Arizona, the records of which exceed 15 years in length. The relation of rainfall frequency to geographic position and topographic relief are discussed, and recommendations are made for the use of these data for design purposes.

INTRODUCTION

In 1935 when the late David L. Yarnell, M. Am. Soc. C. E., published[2] an analysis of rainfall intensity-frequency data, there were only five rainfall-intensity gages in and near the states of New Mexico and Arizona, the records of which were sufficiently long to be used in the analysis. That number represented one intensity gage per 47,000 sq miles. Since then the U. S. Weather Bureau has launched an extensive program for the collection of intensity records in these, as well as in other, states, which eventually will furnish data needed for engineering designs. The data from these newly installed gages have been published consecutively since January, 1940, and have supplied basic information for studies of intensity patterns, but the records are too short for frequency studies.

To help fill the urgent need for detailed recurrence-interval data on rainfall in New Mexico and Arizona, the present study was made of the records of rainfall collected by standard nonrecording gages of the Weather Bureau.

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **July 1, 1943.**

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[2] "Rainfall Intensity—Frequency Data," by David L. Yarnell, *Miscellaneous Publication No. 204*, U. S. D. A., August, 1935.

In addition to data on frequency of heavy rainfalls, it usually is necessary, in the design of certain engineering structures, to assume values of area covered by the design storm and to determine a pattern of rainfall intensity before a hydrograph of runoff can be constructed. With these requisites in view, some data on areal patterns of summer storms and rainfall-intensity records were included to add to already published data pertinent to the adoption of a design storm, with particular reference to a small watershed. Due to the limited areal extent of summer-type rainfall in New Mexico and Arizona, large watersheds usually are capable of producing the largest floods as a result of prolonged rainfall covering large areas, or in these states, a typically winter-type storm.

Storm Types

The normal rainy seasons in the Southwestern States are July to September, inclusive, and December to March. Valley areas usually receive the highest monthly precipitation in July, August, and September, whereas in the high mountain areas the greatest monthly precipitation occurs from December through March, mostly in the form of snow. These seasonal variations in rainfall amounts are related to storm characteristics and sources of moisture.

Winter precipitation generally is the result of frontal activity associated with influx of cold air masses, modified Polar-Pacific or Polar-Continental. Polar-Pacific air picks up moisture from the ocean and moves into the Southwestern States usually as a relatively moist and cool mass that tends to underrun the warmer dry air. If the moist air mass is forced upward over the mountain ranges, continuous showers or nearly steady precipitation in the form of rain or snow is recorded over a period of 48 hr or more.

Modified Polar-Pacific air may move into Arizona and New Mexico from the south, west, or northwest. Invasion from the south or southwest often results in a relatively stationary east-west front which causes widespread, low-intensity rainfall in these states. If the moist air comes from the northwest, heavy snowfall may be recorded on the north rim of the Grand Canyon and on San Francisco peaks, followed by squalls in southeastern Arizona and southwestern New Mexico.

Polar-Continental air usually comes from the northwest and it tends to lose its moisture in northern Utah or Colorado. Such air masses are relatively dry and cold when they reach New Mexico and Arizona, and they do not constitute an important source of moisture in these states.

The Mogollon Rim, a southwest-facing escarpment extending diagonally across central Arizona, receives most of its winter precipitation from the modified Polar-Pacific air masses. This conclusion is borne out by the flood history of the Gila River which drains the Mogollon escarpment and the mountainous country of west-central Arizona and southwestern New Mexico. Generally, high winter rainfalls in the headwaters of the Gila River are preceded by rain along the southern coast of California.

The source of winter precipitation at least partly explains the fact that winter storms generally affect large areas which receive rainfall at low intensities for periods of two or three days or even a week. Although, in a general way, the total winter precipitation increases with elevation, it is influenced to a great extent by the topographic relief.

These characteristics of winter rainfall contrast with those of summer rainfall, which ordinarily is not the direct result of frontal activity. Summer storms usually are of the thunderstorm type with high intensities covering relatively small areas. Ordinarily the moisture is derived originally from the Gulf of Mexico and is carried to Mexico by subtropical easterlies where convection causes the warmth and moisture to rise to high levels over the Mexican Plateau. This air mass is deflected northward and enters the Southwestern States as a moist tongue at high levels, crossing the Mexican border usually west of El Paso, Tex., and east of Yuma, Ariz. Analyses by H. Wexler and J. Namias[3] show that the path of this moist tongue varies from one season to another, and its position is reflected in the summer rainfall in New Mexico and Arizona.

The mechanism by which this vapor is precipitated usually is a combination of convection caused by local heating of the lower layers of air, and orographic influences of topographic relief. Surface heating of the ground and adjacent layers of air during a summer day tends to increase the steepness of the lapse rate (vertical temperature gradient), and convective instability may result in local precipitation. This process usually results in instability above a mountain slope more quickly than above a valley plain, because the temperature of the ground at the two locations increases at about the same rate. Owing to the higher elevation of the point on the mountain, the vertical temperature gradient above that point is greater than that above the valley floor, and it is more probable that rain will fall on the mountain slope than over the valley.

Infrequently, sections of New Mexico and Arizona experience a general summer storm characterized by moderate rainfall over large areas and scattered local areas of heavy precipitation. Whereas the local summer storms are mostly of the convective orographic type, the general storm usually is associated with frontal activity. The effect of topographic relief is relatively small, although together with convective lifting, it may tend to promote local high intensities in the area of general rain. Data on intensities and short-period isohyetal maps of storms of this type are meager. The most adequate discussion of such a storm is the report of the U. S. Engineer Office on the storm of August 13–14, 1940.[4]

Storms of the general type caused the floods of the Pecos and Gila rivers in September, 1941, the Bill Williams River flood of September, 1939, and the San Pedro River flood of September, 1926. In general, the available records indicate that the average intensities for durations of 1 hr or less are considerably less in the general storm than in the local summer storms. In the former type, however, the area covered by intense rainfall tends to be somewhat greater, and the rainfall extends over a longer time period. One of the main points of difference apparently lies in the importance of frontal activity as a causal factor.

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[3] "Mean Monthly Isentropic Charts and Their Relation to Departures of Summer Rainfall," by H. Wexler and J. Namias, *Transactions*, Am. Geophysical Union, Pt. 1, 1938, pp. 164–170.

[4] "Hydrologic Data—Storm of August 13–14, 1940, Arizona and New Mexico," mimeographed September, 1941, Los Angeles Dist., U. S. Engr. Office, Los Angeles, Calif.

Because of the relative infrequence of important storms of the general type, most of the high rainfalls experienced in 24-hr periods in summer and comprising the point-rainfall data analyzed in the present study may be assumed to be of the local storm type. Because of the lack of data, adequate analyses of the area covered by the different storm types are not included in this paper, and thus the recurrence-interval data presented subsequently in Tables 2 and 3 are primarily applicable for design purposes to small watersheds.

AREAL PATTERN OF SUMMER STORMS

The relatively great instability in mountainous country at least partly explains the areal distribution of summer rains in the basin and range topography typical of many parts of New Mexico and Arizona. Rainfall records show that summer rainfall in the valley areas and at the base of the mountains is erratic and spotty, whereas at the crest of the higher mountains the total summer rainfall tends to be similar at all points of a given elevation and tends to increase with elevation. Field observations indicate that in any one summer storm the center of greatest precipitation usually is near the base of the mountain slope.

The areal extent of ordinary summer thunderstorms in New Mexico and Arizona is extremely variable, but the center of high intensity is usually about 5,000 acres. It is important to note, however, that many storms of relatively small area occur nearly simultaneously at widely separated points within an area as large as the State of New Mexico. Storms at any particular location evidently are associated with relatively widespread instability, or the influx of moisture into the entire region. Inspection of any monthly summary of precipitation published in *Climatological Data*[5] shows the regional coincidence of rainfall, but studies of the storms at any given station demonstrate that the storms were separate entities of small areal extent.

There is some evidence that the area covered by rainfall in a given summer storm increases with the total precipitation in the storm center, but only a few good isohyetal maps of individual storms are available. It appears certain that the storms occurring in east-central and southeastern New Mexico in the Pecos River basin cover larger areas than storms in any other section of the two states, and also any given amount of storm rainfall occurs there more frequently than in other sections of these states. This presumably may be attributed to the proximity of the Pecos River basin to the source of summer moisture, the Gulf of Mexico, and to the fact that the Pecos area sometimes is subject to the marginal effects of tropical hurricane disturbances that move into Texas from the Gulf.

Samples of the depth-area data available for local summer storms are presented in Table 1. Storm movement and lack of intensity data for various points within the storm area render this type of depth-area data inadequate for making assumptions of the depth-area-time pattern for a design storm, but in most sections of the Southwestern States no better data than the samples

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[5] *Climatological Data*, published monthly by the U. S. Weather Bureau.

TABLE 1.—AREA, IN SQUARE MILES, COVERED BY VARIOUS QUANTITIES OF RAINFALL (SUMMER STORMS)

No.	Location	Date	Rain, in Inches							
			⋜1	⋜2	⋜3	⋜4	⋜5	⋜6	⋜7	⋜8
71	Pima, Ariz.	September 16, 1939	132	39	8	1.5	...	...	...	...
3	Las Cruces, N. Mex.	August 29, 1935	...	...	...	...	...	38	7.2	2.6
12	Sierra Ancha, Ariz.	August 5, 1939	...	5.2	1.4	0.6	0.2	...	...	...
72	Rodeo, N. Mex.	August 13, 1940	113	11	4	1	...	...	...	...

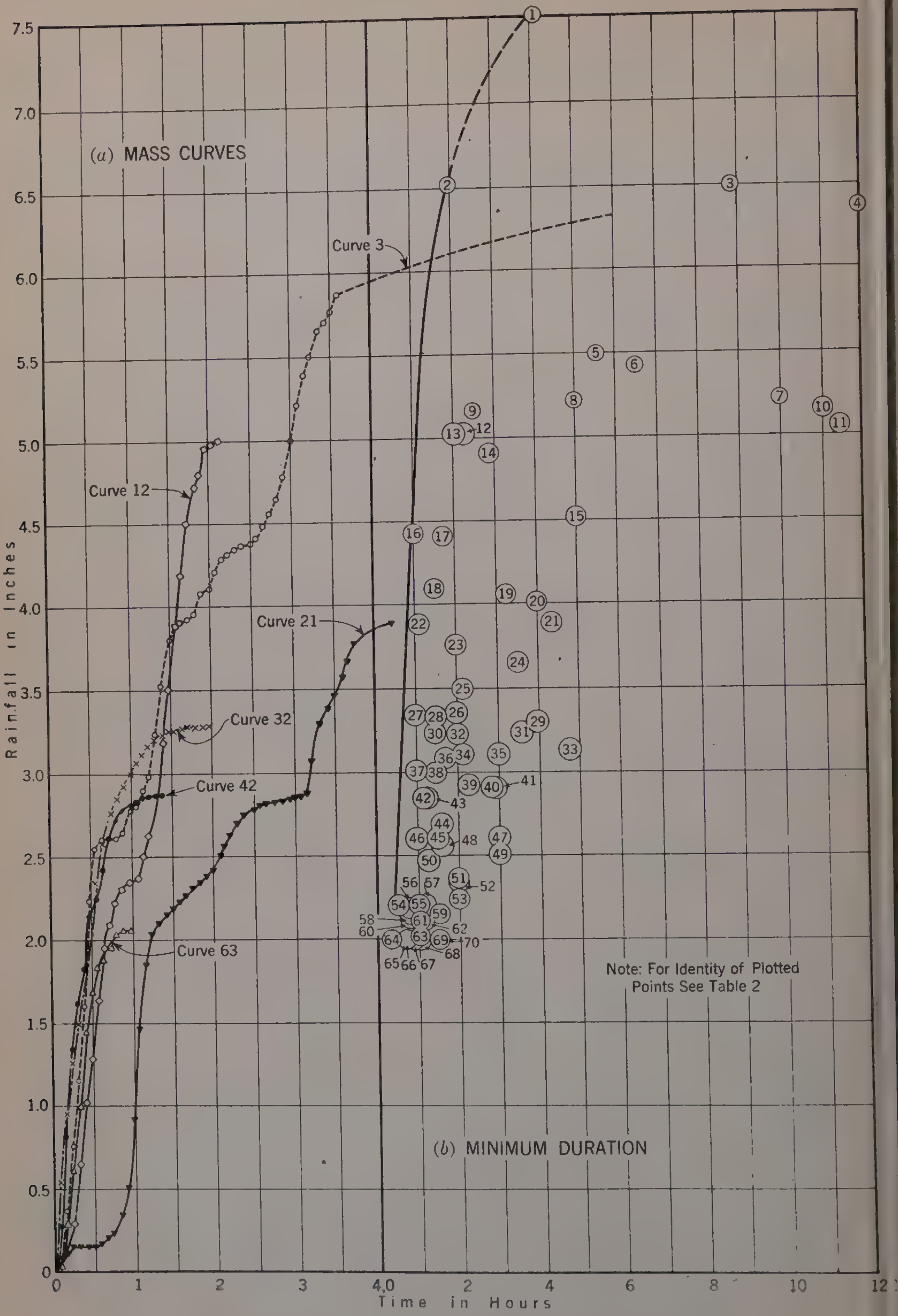
in Table 1 are available. A complete report of depth-area-time relations of storms of these states must await the collection of more records.

RAINFALL INTENSITY

Winter-type storms in New Mexico and Arizona usually produce rainfall at relatively low intensities. Summer rainfalls of the thunderstorm type characteristically are intense and of short duration, with the period of highest intensity at, or soon after, the beginning of rainfall. Except in unusual cases, the periods of high intensity are separated by lulls in the downpour, the period of uninterrupted intense rain lasting less than one-half hour. In practically all cases the intensity decreases more or less gradually to the end of the rain. Mass curves of the most intense rainfalls on record at recording gages in New Mexico and Arizona are plotted in Fig. 1(*a*). Many summer rains have some low intensity fall prior to the most intense portion, but in general the curves in Fig. 1(*a*) are representative of the heaviest rains recorded in the Southwest. It is important to note the general similarity of pattern of mass curves of storms from widely separated localities.

From the records of beginning and ending of rainfall observed at Weather Bureau stations where standard rain gages are used, it is possible to collect additional records of heavy rainfalls of short duration. Fig. 1(*b*) presents the results of a systematic search through the original records of 162 stations in New Mexico, comprising 4,656 station-years and certain additional Arizona records, in an effort to determine the minimum duration of heavy rainfalls. The key to numbered points is given in Table 2. No rain less than 2.00 in. or of durations greater than 14 hr are platted, and many 4-in. rains of duration greater than 6 hr are omitted. The envelope curve drawn in Fig. 1(*b*) indicates the maximum rainfall of a given duration that might reasonably be expected in the Southwestern States.

The mass curves of Fig. 1(*a*) indicate the type of intensity pattern that probably comprised the rainfall totals shown in Fig. 1(*b*), and from these two graphs an approximate rainfall histogram can readily be taken for purposes of hydraulic design of engineering structures. It is important to note that none of the rains platted in Fig. 1(*b*) occurred in midwinter, and relatively few occurred outside the season of summer rains, July to September. It may be surmised from this fact that most of these intense rains covered relatively small areas, typical of summer-type storms.



Note: For Identity of Plotted Points See Table 2

Fig. 1.—Records of Heavy Rainfall in New Mexico and Arizona (From Original Notes of Weather Bureau Observers)

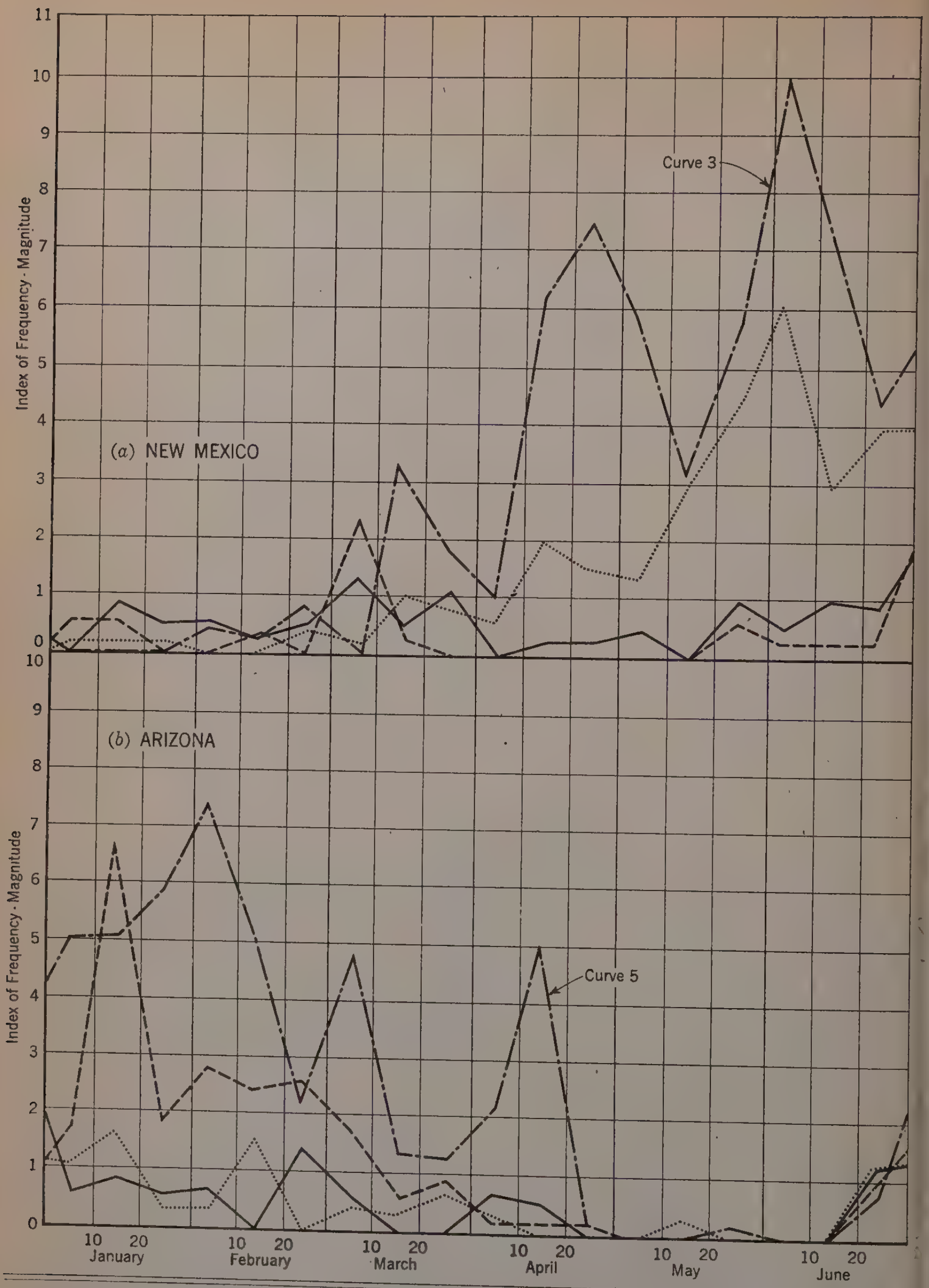
TABLE 2.—KEY TO RAINFALL DATA IN FIG. 1(*b*)

No.	Location	Date
1	Cerro, N. Mex.	August 17, 1922
2	El Paso, Tex.	July 9, 1881
3	Las Cruces, N. Mex.	August 29, 1935
4	Hermosa, N. Mex.	August 30, 1925
5	Artesia, N. Mex.	July 24, 1911
6	Casa Grande Ruin, Ariz.	August 1, 1906
7	Solano, N. Mex.	July 9, 1929
8	Albuquerque, N. Mex.	October 9, 1865
9	Ione, N. Mex.	August 21, 1923
10	White Sands Nat'l Monument, N. Mex.	September 13, 1941
11	Crown King, Ariz.	August 6, 1918
12	Sierra Ancha, Ariz.	August 5, 1939
13	Carnegie Desert Lab., Ariz.	July 2, 1910
14	Crown King, Ariz.	August 11, 1927
15	Tatum, N. Mex.	September 14, 1927
16	Hobbs, N. Mex.	June 7, 1918
17	Elephant Butte, N. Mex.	August 7, 1898
18	Santa Marguerita, Ariz.	August 22, 1935
19	Clovis, N. Mex.	August 4, 1912
20	Diener, N. Mex.	July 3, 1924
21	Las Cruces, N. Mex.	September 21, 1941
22	Clovis, N. Mex.	July 22, 1930
23	Obar, N. Mex.	July 2, 1906
24	Ft. Sumner, N. Mex.	August 13, 1908
25	Corona, N. Mex.	August 14, 1934
26	Mosquero, N. Mex.	May 30, 1938
27	Hope, N. Mex.	August 22, 1908
28	Santa Rosa, N. Mex.	May 30, 1930
29	Roy, N. Mex.	May 30, 1938
30	Elk, N. Mex.	September 25, 1906
31	Ancho, N. Mex.	June 26, 1937
32	Sierra Ancha, Ariz.	September 10, 1933
33	Marinette, Ariz.	August 6, 1939
34	Lake Alice, N. Mex.	August 4, 1935
35	San Marcial, N. Mex.	August 29, 1895
36	Lindrith, N. Mex.	July 30, 1931
37	Hope, N. Mex.	June 5, 1921
38	Hope, N. Mex.	May 5, 1919
39	Carlsbad, N. Mex.	October 12, 1916
40	Torrance, N. Mex.	October 1, 1923
41	Hachita, N. Mex.	August 6, 1931
42	Pima, Ariz. (near)	August 2, 1939
43	Three Rivers, N. Mex.	August 7, 1919
44	Roswell, N. Mex.	September 16, 1923
45	Valmora, N. Mex.	August 6, 1929
46	Duran, N. Mex.	September 16, 1915
47	Bell Ranch, N. Mex.	June 19, 1926
48	Levy, N. Mex.	August 15, 1917
49	Trementina, N. Mex.	August 29, 1916
50	Carlsbad, N. Mex.	July 30, 1916
51	Gallinas R.S., N. Mex.	May 29, 1930
52	Jemez Springs, N. Mex.	July 7, 1923
53	Jemez Springs, N. Mex.	August 3, 1933
54	Lake Alice, N. Mex.	August 30, 1936
55	Roswell, N. Mex.	September 14, 1923
56	Pastura, N. Mex.	August 11, 1933
57	Vermejo Park, N. Mex.	September 9, 1911
58	Camp 23A, Whitlock Valley, Ariz.	September 6, 1939
59	Harvey's Upper Ranch, N. Mex.	July 27, 1919
60	Peoria, Ariz.	August 6, 1939
61	Silver City, N. Mex.	July 20, 1929
62	Mescalero, N. Mex.	August 28, 1923
63	Tucson, Ariz.	July 31, 1935
64	Cuervo, N. Mex.	July 21, 1926
65	Deming, N. Mex.	July 18, 1896
66	Newman, N. Mex.	September 29, 1915
67	Jemez Springs, N. Mex.	July 12, 1910
68	Mineral Hill, N. Mex.	April 23, 1919
69	Laguna, N. Mex.	July 22, 1912
70	Gallinas R.S., N. Mex.	August 16, 1919
71	Pima, Ariz.	September 16, 1939
72	Rodeo, N. Mex.	August 13, 1940

SEASONAL DISTRIBUTION OF HIGH RAINFALLS

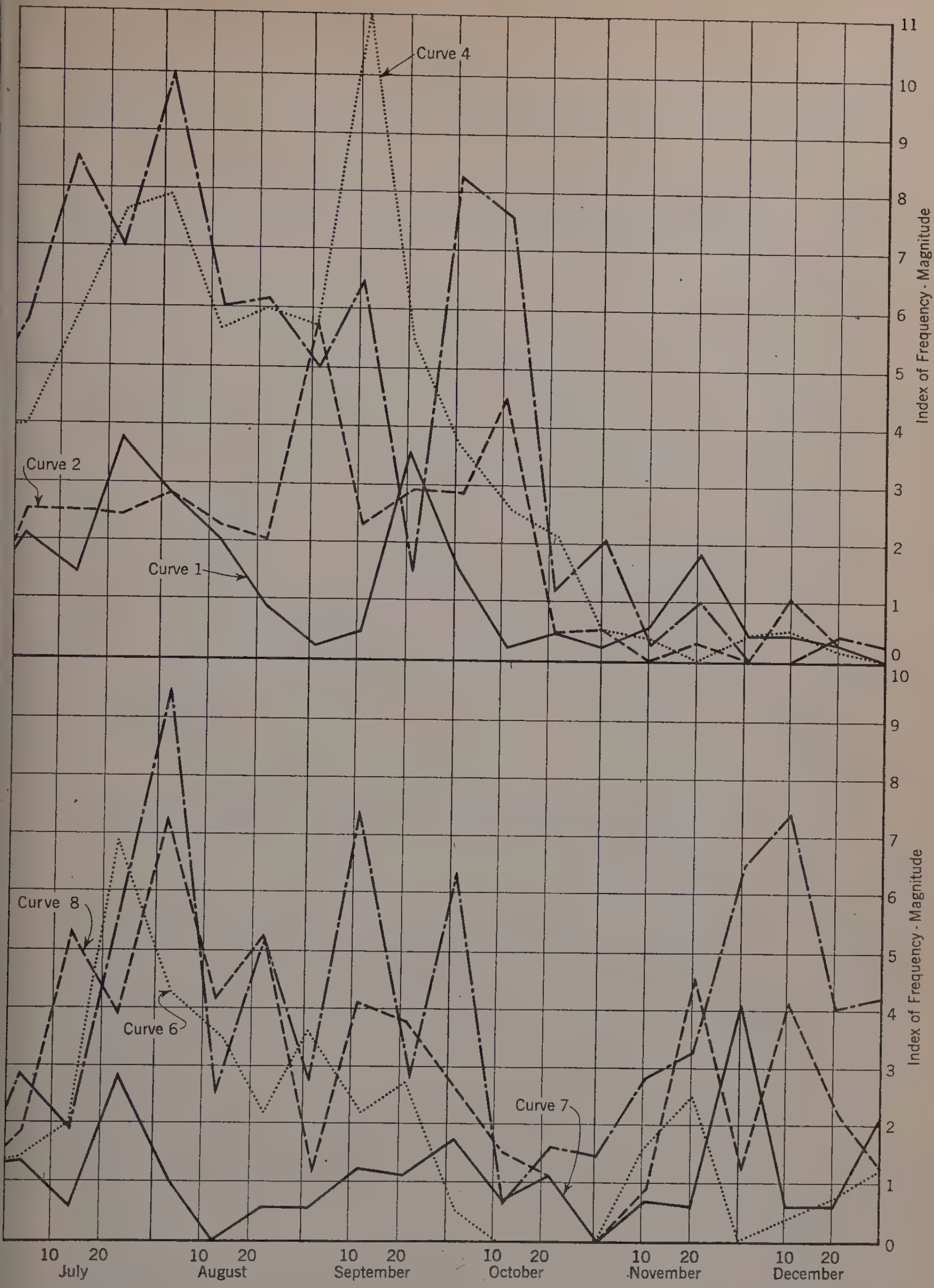
A separate analysis furnishes additional information on the seasonal distribution of high rainfalls. A tabulation was made of all rainfalls exceeding 2.00 in. in a 1-day period for 268 stations in the two states, comprising a total of 7,048 station-years of record. Stations were grouped into eight general regions by dividing each state approximately into four quadrants. For each region, rainfall depths were platted against the day of the year on which each occurred. An index was thus derived that weights both the frequency and magnitude of high rains, and is platted for the four regions in New Mexico and four regions in Arizona in Fig. 2.

The index was derived by dividing the year into periods of ten consecutive days, and the rainfall magnitude into categories of half inches, beginning with 2.00 to 2.50 in., 2.50 to 3.00 in., and so forth. For a given 10-day period including all stations, the number of rainfalls that fell in each magnitude category was multiplied by the average value of the magnitude category, and the sum of the products times 100, divided by the station-years of record for the region, was used as the index of magnitude-frequency. For example, in the upper Gila River and southeastern Arizona area, 35 stations were used, comprising 976 station-years of record. Between January 1 and January 10, there were five rains between 2.00 in. and 2.50 in. in magnitude, and one rain between



	(a) New Mexico; Number of:	Stations	Station-years
1	Upper Rio Grande watershed and northwestern New Mexico	37	925
2	Lower Rio Grande watershed and southwestern New Mexico	25	802
3	Canadian River watershed and northeastern New Mexico	39	1,068
4	Pecos River watershed and southeastern New Mexico	47	1,227

Fig. 2.—Seasonal Variation of Rainfall Frequency (Magnitude



	(b) Arizona; Number of:	Stations	Station-years
5	Verde River watershed and northwestern Arizona	43	988
6	Lower Gila River watershed and southwestern Arizona	27	671
7	Little Colorado River watershed and northeastern Arizona	15	391
8	Upper Gila River watershed and southeastern Arizona	35	976

Index Derived from Rains Exceeding 2.00 In. in One Day)

5.50 in. and 6.00 in. The index platted at the midpoint of the period January 1 to 10 was $100 \times \frac{(5 \times 2.25) + (1 \times 5.75)}{976} = 1.74$.

It was necessary to divide by the number of station-years of record to make the computed indexes of the various regions comparable for platting, and the multiplication by the arbitrary value 100 was simply for the purpose of making the platted indexes whole numbers to facilitate platting and comparison.

The platted indexes in Fig. 2 show that the eastern section of New Mexico experiences high rainfalls more frequently than the western section of the state; but all sections receive high rainfalls predominantly in the summer and fall seasons, whereas November through March is comparatively free of heavy rainfalls. The heaviest rainfalls in western New Mexico occur between the first of July and the middle of August, and again in mid-September.

In Arizona, however, all regions except the northeast quadrant have a high index in summer, whereas in winter only the southeast and northwest quadrants have high indexes. These differences can be explained by the relation of topography, geographic position, and source of moisture. It was stated previously that summer moisture originates in the Gulf of Mexico and moves into New Mexico and Arizona primarily from the south, and that the Pecos Area in southeastern New Mexico sometimes is affected by tropical disturbances that cause great rainfalls in southern and central Texas. It may be expected, therefore, that the eastern section of the state would experience more high rains than the western because of the proximity to the source of moisture.

The southwestern section of New Mexico has a higher frequency-magnitude index than the northwest, since the latter is in the rain shadow of the mountainous southwestern section.

The index for regions in Arizona in summer is highest in the mountainous southeast and northwest quadrants, which can be related to the effect of topography. The index for southwestern Arizona is high in midsummer—July and August—because of the convective storms that are common over the desert at that season. Northeastern Arizona is on the plateau in the rain shadow of the Mogollon Rim and the mountainous central belt, and probably for this reason the index is low.

In winter, when moisture is derived from the Pacific Ocean and moves in primarily from the Southwest, the frequency-magnitude index should be high only for the mountainous sections of Arizona, the southeast and northwest quadrants, and relatively low in other areas and in New Mexico. The present study supports this hypothesis.

FREQUENCY OF HIGH RAINFALLS

Recurrence-interval curves were drawn for each standard rain-gage station in New Mexico and Arizona whose records exceed 15 years, and for certain stations of shorter record, in order to derive values of 24-hr rainfall that may be expected to be equaled or exceeded in a given period of years. These data are useful in the design of engineering structures, especially where it is necessary to design for a rain greater than has been experienced in the past. The data also indicate the recurrence-interval of rainfalls that have occurred in the past.

In this analysis the highest one-day rainfall in each year of record at the station was tabulated, the data arranged in order of magnitude, and the curve platted by the "exceedance interval," or "California," method. From the platted curves, the 24-hr rainfall that probably will be equaled or exceeded in 5, 10, 15, 20, 35, 50, 75, and 100 years, was tabulated and the data are presented in Table 3(*a*) for New Mexico stations and Table 3(*b*) for Arizona stations.

The rainfall data used were recorded by standard nonrecording rain gages maintained by the Weather Bureau. The rainfall depths were taken from the original records on which, in about half the cases, the observer had noted the time of beginning and time of ending of the precipitation. By use of these notations of times it was possible to ascertain the greatest rainfall in a 24-hr period, which was the value used in the frequency array wherever possible. Particularly in the case of summer rain, the precipitation often starts in mid-afternoon and ends during the night, which is a period of less than 24 hr. Many observers read the gage at sunset; so, in the foregoing example, if no rain fell on the day following, the sum of the two sunset readings equaled the amount that fell in a 24-hr period. Rains continuing through two or more days could not be broken down accurately to determine the largest amount in a 24-hr period and, therefore, the largest amount recorded for one day was used in the recurrence-interval array. The method used approximates, closely, the actually greatest fall in a 24-hr period, since a large percentage of the high rains fell in summer and lasted less than 24 hr.

The characteristics of summer and non-summer precipitation were considered sufficiently distinct and important to warrant separation in the recurrence-interval determinations. Arrays were made separately for the highest rainfall recorded each year between June 1 and September 30 (the summer period), and the highest rain recorded in each complete year. Certain stations may be expected to experience the same 24-hr rainfall at a given recurrence-interval in summer as they will experience in the entire year as can be seen in the tabulations of 50, 75, and 100-yr frequencies. This simply indicates that that station characteristically received the largest 24-hr rainfalls in the summer period and, therefore, the recurrence-interval curves for summer and all year would be practically identical.

Winter values of rainfall amounts probably are somewhat less accurate than summer values owing to the effect of snow. One source of error was the failure of a gage to catch the full snowfall, but probably a more important error was introduced by the general policy of assuming that 1 in. of snow equaled 0.10 in. of precipitation in the form of water. Investigations by the Weather Bureau indicate that, in particular instances, much larger amounts of precipitation were assumed by the use of this policy than actually fell.

The compilation of a recurrence-interval array necessarily involves some exercise of judgment. In many cases, particular months were missing from a

TABLE 3.—FREQUENCY-
Data from Records Pub-

No.	Station	Elevation (feet)	Average annual precipitation through 1939 (inches)	Length of record (years)	RECORD USED FOR FREQUENCY ARRAY		AVERAGE OF MAXIMUM 1-DAY RAIN EACH:	
					Dates (inclusive)	Number years	Summer[1]	Year[2]
								(*a*) NEW
1	Dulce	6,767	18.13	34	1906–1915; 1917–1939	33	1.16	1.34
2	Aztec	5,590	9.06	38	1895–1901; 1911–1916; 1918–1923; 1925–1939	34	.91	1.01
3	Fruitland	5,165	6.68	35	1893; 1895; 1897; 1904–1915; 1917–1922; 1939	22	.79	.95
4	Farmington	5,220	8.35	25	1914–1930; 1933–1939	24	.87	.94
5	Bloomfield	5,500	8.79	41	1905–1939	35	.90	.97
6	Haynes	6,600	11.80	16	1910–1917; 1920–1926	15	1.16	1.20
7	Tohatchi	6,800	11.07	24	1915; 1918; 1923; 1926–1927; 1929–1939	16	1.13	1.19
8	Crownpoint	6,800	11.15	26	1914–1919; 1921–1939	25	1.05	1.11
9	Gamerco	6,785	11.94	22	1922–1939	18	.91	1.12
10	Ft. Wingate	6,997	14.45	48	1897–1910	14	1.23	1.44
11	McGaffey Ranger Station	8,000	17.77	27	1916–1939	24	1.10	1.41
12	Bluewater	6,732	10.28	34	1897–1902; 1909–1917; 1926–1939	29	1.06	1.20
13	Ramah	7,200	13.68	15	1922; 1924; 1927–1939	15	1.36	1.68
14	San Fidel	6,100	10.37	20	1920–1939	20	1.20	1.22
15	Black Rock	6,455	12.29	31	1909–1910; 1912; 1914–1939	29	1.01	1.10
16	Chama	7,851	22.10	44	1893–1896; 1905–1918; 1921–1939	37	1.07	1.40
17	Cerro	7,668	13.98	29	1910–1916; 1918–1930; 1932–1939	28	1.20	1.33
18	Red River Canyon	8,956	21.65	34	1906–1907; 1910–1939	32	1.02	1.19
19	Tres Piedras	8,076	16.05	40	1905–1910; 1912–1914; 1916–1939	33	1.11	1.28
20	Aspen Grove Ranch	9,000	23.38	31	1897–1898; 1910–1939	32	1.10	1.40
21	Eagle Rock Ranger Station	8,000	15.70	17	1914–1930	17	1.07	1.22
22	Bateman's Ranch	8,900	23.48	31	1910–1939	30	1.24	1.46
23	Taos Canyon	8,959	19.52	31	1909–1939	31	1.04	1.22
24	Taos	6,983	12.61	47	1893–1895; 1902–1939	41	.93	1.08
25	Truchas	7,935	15.55	31	1905; 1909; 1910–1916; 1918–1939	31	1.14	1.29
26	Espanola	5,590	9.96	35	1895–1900; 1902; 1905; 1907–1909; 1911–1918; 1920–1929; 1938–1939	31	1.04	1.14
27	Selsor Ranch	8,500	22.19	28	1894; 1912–1926; 1928–1939	28	1.30	1.46
28	Alamos Ranch	7,328	18.13	29	1911–1916; 1918–1920; 1922–1939	27	1.34	1.57
29	Frijoles Canyon	6,200	15.38	16	1924–1939	16	1.54	1.66
30	Jemez Springs	6,100	18.07	30	1910–1939	30	1.49	1.63
31	Lee's Ranch	8,691	22.13	16	1924–1939	16	1.89	2.08
32	Santa Fe Canyon	8,100	17.69	13	1910–1916; 1923–1928	13	1.14	1.26
33	Santa Fe	6,994	14.39	90	1850; 1853–1861; 1863–1866; 1868–1882; 1885–1936; 1938–1939	83	1.15	1.29
34	Stanley	6,317	12.20	29	1909–1913; 1916–1926; 1929–1939	27	1.32	1.39
35	Albuquerque	5,314	8.18	69	1850–1854; 1856–1861; 1864–1865; 1879; 1889–1890; 1892–1908; 1910–1939	63	.95	1.13
36	Laguna	5,840	11.14	30	1905–1914; 1919–1921; 1927–1934; 1936–1939	25	1.27	1.48
37	Estancia	6,300	15.14	26	1905–1907; 1909–1911; 1913–1917; 1923–1927; 1937–1939	19	1.21	1.35
38	McIntosh	6,355	13.04	12	1928–1939	12	1.26	1.53
39	Los Lunas	4,900	8.46	51	1893–1897; 1899–1901; 1903–1909; 1911–1939	44	1.16	1.31
40	Tajique	7,100	19.68	20	1910–1918; 1920–1939	29	1.47	1.73
41	Mountainair	6,475	16.03	33	1902–1915; 1917; 1919–1921; 1923; 1928–1939	31	1.45	1.70
42	Datil	7,850	13.22	28	1906–1908; 1916–1924; 1926–1939	26	1.19	1.27
43	Magdalena	6,556	12.68	37	1905–1915; 1918–1925; 1927–1931; 1934–1939	30	1.14	1.31
44	Socorro	4,600	10.19	53	1893–1894; 1896–1897; 1899–1901; 1904–1939	43	1.19	1.38
45	San Marcial	4,447	9.11	83	1895–1899; 1901; 1904–1909; 1912–1919; 1921–1939	39	1.17	1.38
46	Glorieta Ranch	6,300	11.88	19	1910–1928	18	1.10	1.40
47	Chloride Ranger Station	6,500	14.84	23	1922–1939	18	1.26	1.49

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[1] Average of the maximum 1-day rain recorded for the
[2] Average of the maximum 1-day rain recorded in each

MAGNITUDE OF DEPRECIATION
lished by U. S. Weather Bureau)

ONE-DAY RAINFALL WHICH PROBABLY WILL BE EQUALED OR EXCEEDED ONCE IN:																No.
5 years		10 years		15 years		20 years		35 years		50 years		75 years		100 years		
Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	
MEXICO																
1.33	1.65	1.75	2.20	2.06	2.60	2.30	2.90	2.91	3.65	3.40	4.25	4.00	5.00	4.50	5.60	1
1.25	1.35	1.48	1.59	1.60	1.70	1.69	1.80	1.83	1.98	1.92	2.09	2.02	2.19	2.10	2.28	2
1.13	1.24	1.40	1.51	1.58	1.66	1.70	1.78	1.96	2.00	2.12	2.18	2.34	2.35	2.50	2.50	3
1.21	1.29	1.51	1.58	1.69	1.71	1.80	1.82	2.02	2.05	2.20	2.20	2.38	2.38	2.50	2.50	4
1.30	1.30	1.49	1.50	1.57	1.58	1.60	1.61	1.65	1.70	1.68	1.71	1.68	1.75	1.70	1.78	5
1.73	1.73	2.05	2.05	2.20	2.20	2.31	2.31	2.50	2.50	2.65	2.65	2.77	2.77	2.86	2.86	6
1.70	1.70	2.31	2.31	2.70	2.70	3.05	3.05	3.78	3.78	4.30	4.30	4.95	4.95	5.50	5.50	7
1.48	1.50	1.71	1.72	1.82	1.85	1.91	1.92	2.05	2.08	2.16	2.16	2.23	2.23	2.30	2.30	8
1.38	1.70	1.60	2.01	1.70	2.16	1.78	2.22	1.89	2.35	1.92	2.50	1.98	2.47	2.00	2.50	9
1.50	1.86	1.78	2.28	1.93	2.53	2.04	2.72	2.28	3.14	2.43	3.42	2.60	3.77	2.70	4.00	10
1.58	1.88	1.90	2.25	2.05	2.46	2.17	2.60	2.37	2.92	2.50	3.11	2.63	3.36	2.71	3.50	11
1.40	1.64	1.68	2.01	1.81	2.25	1.92	2.41	2.12	2.77	2.28	3.00	2.40	3.30	2.50	3.50	12
2.10	2.40	2.55	2.75	2.73	2.91	2.85	3.05	3.10	3.22	3.25	3.35	3.41	3.48	3.50	3.50	13
1.54	1.70	1.68	1.92	1.72	2.06	1.78	2.15	1.81	2.30	1.85	2.40	1.88	2.50	1.89	2.55	14
1.40	1.51	1.78	1.89	1.98	2.08	2.11	2.21	2.42	2.50	2.67	2.69	2.90	2.90	3.10	3.10	15
1.46	1.70	1.74	2.08	1.89	2.33	1.99	2.54	2.17	3.00	2.28	3.32	2.40	3.75	2.50	4.10	16
1.42	1.62	2.00	2.22	2.42	2.65	2.79	3.00	3.58	3.74	4.20	4.30	4.95	5.00	5.60	5.60	17
1.31	1.51	1.60	1.78	1.75	1.90	1.88	2.00	2.10	2.20	2.29	2.31	2.48	2.48	2.60	2.60	18
1.50	1.60	1.81	1.89	2.00	2.06	2.12	2.20	2.41	2.46	2.60	2.62	2.83	2.85	3.00	3.00	19
1.48	1.96	1.72	2.42	1.82	2.70	1.91	2.90	2.05	3.31	2.16	3.61	2.26	3.90	2.31	4.12	20
1.40	1.49	1.57	1.61	1.63	1.69	1.70	1.72	1.80	1.81	1.87	1.89	1.92	1.92	1.99	1.99	21
1.62	1.85	1.88	2.14	1.99	2.30	2.08	2.40	2.21	2.62	2.31	2.78	2.40	2.90	2.50	3.00	22
1.33	1.55	1.55	1.69	1.65	1.78	1.71	1.82	1.85	1.92	1.92	2.00	2.01	2.05	2.10	2.10	23
1.30	1.39	1.48	1.55	1.53	1.61	1.59	1.65	1.66	1.70	1.70	1.74	1.74	1.78	1.78	1.80	24
1.66	1.91	2.05	2.24	2.22	2.37	2.32	2.46	2.51	2.59	2.62	2.69	2.75	2.75	2.82	2.82	25
1.44	1.50	1.65	1.71	1.72	1.83	1.79	1.91	1.88	2.02	1.91	2.11	1.98	2.20	2.00	2.28	26
1.70	1.95	2.00	2.22	2.18	2.36	2.30	2.48	2.52	2.68	2.70	2.80	2.89	2.93	3.00	3.05	27
1.80	2.03	2.18	2.59	2.40	2.90	2.58	3.18	2.91	3.70	3.19	4.11	3.46	4.60	3.68	4.95	28
2.18	2.18	2.62	2.62	2.80	2.80	3.00	3.00	3.25	3.35	3.46	3.60	3.65	3.89	3.80	4.10	29
2.01	2.19	2.38	2.61	2.51	2.80	2.64	2.97	2.83	3.25	2.99	3.41	3.10	3.63	3.20	3.80	30
2.72	2.85	3.04	3.21	3.20	3.40	3.30	3.51	3.43	3.79	3.52	3.90	3.60	4.10	3.70	4.20	31
1.58	1.63	1.80	1.86	1.91	1.98	2.00	2.07	2.19	2.21	2.30	2.30	2.40	2.40	2.50	2.50	32
1.52	1.60	1.83	1.97	2.05	2.22	2.21	2.48	2.58	2.98	2.81	3.39	3.13	3.95	3.40	4.50	33
1.79	1.90	2.17	2.23	2.40	2.41	2.57	2.59	2.85	2.85	3.09	3.09	3.33	3.33	3.50	3.50	34
1.30	1.50	1.89	2.14	2.28	2.63	2.60	3.02	3.35	3.95	3.89	4.69	4.50	5.45	5.00	6.00	35
1.68	1.90	1.88	2.29	1.94	2.50	1.99	2.69	2.06	3.00	2.10	3.28	2.13	3.53	2.17	3.75	36
1.80	1.84	2.15	2.15	2.35	2.35	2.45	2.45	2.71	2.71	2.90	2.90	3.10	3.10	3.21	3.21	37
1.62	1.93	2.02	2.31	2.30	2.58	2.50	2.78	2.93	3.12	3.25	3.40	3.60	3.70	3.89	3.90	38
1.61	1.78	1.91	2.04	2.01	2.15	2.10	2.21	2.23	2.35	2.31	2.41	2.40	2.50	2.48	2.57	39
1.98	2.18	2.38	2.56	2.60	2.79	2.80	2.97	3.18	3.30	3.45	3.52	3.71	3.79	4.00	4.00	40
1.95	2.18	2.29	2.49	2.46	2.65	2.61	2.78	2.90	2.99	3.11	3.15	3.30	3.35	3.43	3.49	41
1.51	1.58	1.71	1.73	1.81	1.82	1.90	1.90	2.02	2.02	2.11	2.11	2.21	2.21	2.30	2.30	42
1.50	1.78	1.70	2.07	1.78	2.19	1.82	2.29	1.90	2.44	1.95	2.57	1.98	2.68	2.00	2.76	43
1.53	1.87	2.05	2.59	2.41	3.10	2.71	3.51	3.37	4.50	3.87	5.12	4.50	6.20	5.00	7.00	44
1.70	1.90	2.40	2.62	2.90	3.11	3.35	3.50	4.30	4.44	5.00	5.10	6.00	6.00	6.70	6.70	45
1.43	1.70	1.69	1.90	1.83	2.02	1.95	2.11	2.15	2.30	2.30	2.40	2.45	2.53	2.58	2.60	46
1.65	1.83	1.81	2.21	1.90	2.48	1.98	2.69	2.07	3.03	2.12	3.30	2.20	3.60	2.24	3.80	47

period June 1–September 30 in each year of record.
year of record.

TABLE 3.—

No.	Station	Elevation (feet)	Average annual precipitation through 1939 (inches)	Length of record (years)	RECORD USED FOR FREQUENCY ARRAY		AVERAGE OF MAXIMUM 1-DAY RAIN EACH:	
					Dates (inclusive)	Number years	Summer[1]	Year[2]
							(*a*) NEW MEXICO—	
48	Hermosa	7,200	17.79	15	1914–1926	13	1.75	2.18
49	Elephant Butte	4,265	9.91	55	1895–1904; 1908–1916; 1918–1921; 1923–1939	40	1.38	1.42
50	Kingston	6,500	18.18	25	1916–1939	24	1.59	1.72
51	Hillsboro	5,236	12.61	34	1896–1900; 1904–1909; 1911; 1925–1933; 1935–1939	26	1.48	1.54
52	Garfield	4,100	10.09	33	1894–1899; 1905–1917; 1920–1923; 1925; 1927	25	1.24	1.27
53	Lake Valley	5,412	13.41	33	1905–1923; 1927–1939	32	1.52	1.59
54	Jornada Experimental Range	4,150	9.28	26	1914–1939	26	1.05	1.14
55	Deming	4,331	9.66	64	1893–1939	47	1.15	1.23
56	Las Cruces (Agricultural College)	3,909	8.66	83	1892–1939	48	1.11	1.17
57	Cambray	4,225	8.92	39	1899–1933; 1936–1939	39	1.16	1.28
58	Newman	3,989	9.05	24	1909–1932	24	1.29	1.35
59	Lanark	4,156	8.20	25	1899–1918; 1921	21	1.14	1.18
60	Columbus	4,054	9.58	31	1909–1939	31	1.25	1.32
61	Horse Springs	7,070	13.79	31	1900–1902; 1906–1907; 1909–1921; 1923–1927; 1930–1938	32	1.03	1.20
62	Luna Ranger Station	7,300	16.18	40	1903–1906; 1908–1918; 1920–1939	35	1.09	1.18
63	Hood Ranger Station	5,838	14.19	23	1916–1939	24	1.01	1.24
64	Alma	4,800	14.89	26	1894–1901; 1903–1905; 1910–1925	27	1.12	1.34
65	Cliff (Gila)	4,470	14.22	32	1895–1900; 1905–1910; 1912–1914; 1916; 1931–1939	25	1.44	1.61
66	Pinos Altos	7,000	22.89	29	1911–1932; 1934–1939	28	1.48	1.65
67	Mimbres	6,250	18.33	26	1905–1919; 1930–1939	25	1.35	1.54
68	Ft. Bayard	6,152	15.70	71	1897–1919; 1921–1939	42	1.38	1.54
69	Silver City	5,937	17.38	37	1901; 1912–1939	29	1.46	1.52
70	Redrock	4,150	12.30	35	1905–1939	35	1.32	1.53
71	Lordsburg	4,245	9.57	59	1893–1939	47	1.10	1.26
72	Hachita	4,504	10.76	30	1909–1939	31	1.34	1.44
73	Rodeo	4,118	11.04	31	1909–1939	31	1.29	1.48
74	Lake Alice	6,950	19.85	31	1909–1911; 1913–1939	30	1.48	1.86
75	Vermejo Park	7,600	17.13	19	1904–1920; 1924–1925	19	1.25	1.38
76	Raton	6,660	15.61	44	1895–1905; 1909; 1912–1939	40	1.75	1.98
77	Des Moines	6,622	17.23	24	1916–1939	24	1.64	1.79
78	Dawson	6,396	15.54	31	1909–1939	31	1.44	1.76
79	Maxwell	5,894	14.53	27	1905; 1908–1931	25	1.40	1.60
80	Cimarron	6,427	15.56	36	1904–1939	36	1.30	1.74
81	Clayton	5,049	15.62	35	1896–1899; 1909–1911; 1913–1939	34	1.49	1.78
82	Springer	5,857	14.18	51	1893–1894; 1896–1897; 1900–1901; 1904–1939	42	1.44	1.59
83	Miami	6,000	15.73	32	1908–1917; 1919–1926; 1928–1939	30	1.47	1.68
84	Taylor	5,661	15.57	24	1909–1932	24	1.20	1.35
85	Pasamonte	5,650	15.11	30	1910–1939	30	1.57	1.71
86	Black Lake	8,348	16.21	31	1909–1939	31	1.33	1.46
87	Aurora	8,849	20.45	31	1909–1939	31	1.45	1.77
88	Vance (Thomas)	4,825	15.28	29	1911–1939	29	1.57	1.82
89	Abbott	5,771	14.98	32	1909–1939	31	1.59	1.82
90	Chacon	8,510	20.38	31	1909–1939	31	1.20	1.39
91	Levy	6,252	17.24	30	1909–1918; 1922–1939	28	1.68	1.86
92	Palo Verde	5,880	15.97	29	1912–1939	28	1.39	1.86
93	Roy	5,884	15.18	31	1909–1939	31	1.55	1.92
94	Ft. Union	6,885	18.01	62	1895; 1897–1918	23	1.71	1.87
95	Solano	5,622	16.71	31	1909–1939	31	1.89	2.14
96	Hoosier Ranch	5,680	15.40	29	1911–1939	29	1.80	2.29
97	Valmora	6,200	16.51	22	1917–1923; 1926–1939	21	1.52	1.67
98	Mosquero	5,579	16.02	25	1915–1939	25	1.78	1.96
99	Ione	4,850	16.57	30	1916–1939	24	2.19	2.40
100	Bell Ranch	5,400	15.24	41	1899–1939	41	1.62	1.91
101	Obar	4,225	16.36	34	1906–1911; 1914–1923; 1926–1939	30	1.80	2.13
102	Trementina	5,000	15.68	22	1908–1918; 1920–1928	20	1.52	1.91
103	Logan	3,851	16.29	34	1906–1920; 1922–1937; 1939	32	1.85	1.91
104	Tucumcari No. 1	4,100	16.35	35	1905–1939	35	1.74	2.05

[1] Average of the maximum 1-day rain recorded for the

[2] Average of the maximum 1-day rain recorded in each

(Continued)

ONE-DAY RAINFALL WHICH PROBABLY WILL BE EQUALED OR EXCEEDED ONCE IN:																No.
5 years		10 years		15 years		20 years		35 years		50 years		75 years		100 years		
Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	
(Continued)																
2.25	3.10	2.95	3.85	3.41	4.28	3.80	4.55	4.55	5.15	5.10	5.50	5.80	5.95	6.30	6.30	48
1.75	1.81	2.29	2.38	2.67	2.70	2.95	3.00	3.60	3.60	4.06	4.06	4.60	4.60	5.00	5.00	49
2.10	2.30	2.52	2.73	2.80	2.97	2.99	3.13	3.35	3.50	3.64	3.78	3.95	4.00	4.20	4.20	50
1.98	2.00	2.32	2.34	2.55	2.55	2.68	2.68	2.97	2.97	3.12	3.12	3.34	3.34	3.50	3.50	51
1.70	1.71	1.91	1.91	2.00	2.00	2.05	2.05	2.15	2.15	2.20	2.20	2.26	2.26	2.30	2.30	52
1.90	1.98	2.49	2.51	2.90	2.95	3.22	3.29	4.00	4.03	4.60	4.60	5.40	5.40	6.00	6.00	53
1.40	1.53	1.76	1.81	1.96	2.00	2.12	2.17	2.45	2.46	2.68	2.71	2.95	2.97	3.20	3.20	54
1.65	1.70	1.84	1.91	1.92	1.99	1.98	2.01	2.02	2.08	2.05	2.10	2.06	2.13	2.08	2.12	55
1.45	1.55	1.99	2.09	2.36	2.46	2.68	2.76	3.35	3.40	3.88	3.98	4.50	4.58	5.00	5.00	56
1.59	1.74	1.89	2.01	2.04	2.11	2.13	2.21	2.35	2.39	2.49	2.51	2.60	2.62	2.70	2.70	57
1.63	1.66	1.88	1.90	2.00	2.03	2.12	2.15	2.33	2.35	2.47	2.50	2.60	2.63	2.70	2.70	58
1.65	1.65	2.02	2.02	2.30	2.30	2.44	2.44	2.80	2.80	3.05	3.05	3.35	3.35	3.58	3.58	59
1.75	1.80	2.20	2.21	2.48	2.50	2.68	2.69	3.10	3.10	3.40	3.40	3.75	3.75	4.00	4.00	60
1.33	1.55	1.50	1.76	1.60	1.88	1.66	1.98	1.80	2.10	1.89	2.20	1.97	2.30	2.01	2.38	61
1.41	1.68	1.68	1.92	1.82	2.06	1.94	2.19	2.18	2.35	2.32	2.50	2.50	2.64	2.62	2.76	62
1.35	1.52	1.53	1.70	1.62	1.80	1.68	1.89	1.78	2.03	1.84	2.11	1.90	2.20	1.92	2.29	63
1.40	1.68	1.59	1.92	1.69	2.08	1.78	2.18	1.90	2.37	1.99	2.50	2.08	2.65	2.12	2.78	64
1.81	2.10	2.19	2.38	2.31	2.50	2.41	2.59	2.64	2.75	2.79	2.85	2.90	2.95	3.00	3.01	65
1.90	2.05	2.15	2.30	2.30	2.40	2.41	2.50	2.60	2.69	2.78	2.80	2.90	2.90	3.00	3.00	66
1.78	2.00	2.08	2.29	2.25	2.41	2.38	2.52	2.60	2.70	2.75	2.82	2.90	2.98	3.00	3.09	67
1.80	1.98	2.21	2.39	2.49	2.62	2.70	2.81	3.10	3.20	3.41	3.50	3.80	3.80	4.00	4.00	68
1.90	1.92	2.11	2.13	2.22	2.25	2.30	2.31	2.38	2.45	2.42	2.51	2.48	2.60	2.50	2.68	69
1.70	2.00	1.92	2.40	2.05	2.60	2.13	2.79	2.29	3.09	2.39	3.30	2.40	3.55	2.58	3.70	70
1.43	1.61	1.77	1.98	1.98	2.18	2.11	2.34	2.43	2.68	2.63	2.90	2.88	3.40	3.05	3.40	71
1.94	2.07	2.39	2.50	2.60	2.75	2.79	2.93	3.10	3.28	3.32	3.50	3.60	3.78	3.80	4.00	72
1.80	2.03	2.13	2.32	2.30	2.45	2.41	2.52	2.60	2.68	2.70	2.79	2.83	2.85	2.90	2.90	73
1.90	2.40	2.28	2.97	2.50	3.30	2.66	3.53	3.00	4.10	3.21	4.43	3.50	4.85	3.70	5.10	74
1.73	1.90	2.01	2.17	2.20	2.30	2.31	2.40	2.55	2.60	2.70	2.72	2.88	2.88	3.00	3.00	75
2.40	2.68	3.19	3.40	3.65	3.85	4.01	4.20	4.85	4.95	5.48	5.59	6.20	6.30	6.80	6.80	76
2.45	2.65	3.00	3.10	3.25	3.30	3.40	3.45	3.63	3.65	3.78	3.80	3.90	3.95	4.00	4.00	77
1.88	2.26	2.38	2.71	2.68	3.00	2.91	3.21	3.43	3.65	3.80	4.00	4.25	4.35	4.60	4.60	78
1.90	2.24	2.37	2.78	2.62	3.02	2.83	3.20	3.25	3.50	3.51	3.70	3.85	3.90	4.10	4.10	79
1.75	2.32	2.31	2.95	2.70	3.35	3.00	3.60	3.60	4.12	4.08	4.49	4.60	4.85	5.00	5.09	80
1.95	2.31	2.45	2.90	2.73	3.30	2.99	3.59	3.45	4.20	3.80	4.70	4.20	5.20	4.50	5.60	81
1.82	2.02	2.28	2.43	2.50	2.70	2.71	2.89	3.15	3.25	3.48	3.50	3.80	3.80	4.00	4.00	82
1.90	2.18	2.20	2.60	2.35	2.88	2.47	3.09	2.68	3.50	2.81	3.80	2.93	4.13	3.00	4.30	83
1.65	1.75	1.88	1.93	1.95	2.00	2.00	2.05	2.09	2.11	2.12	2.17	2.18	2.18	2.20	2.20	84
2.21	2.36	2.61	3.00	2.80	3.35	2.93	3.60	3.18	4.18	3.31	4.59	3.48	5.00	3.60	5.38	85
1.80	1.92	2.03	2.10	2.10	2.16	2.15	2.20	2.22	2.25	2.27	2.29	2.29	2.29	2.30	2.30	86
1.90	2.48	2.49	3.04	2.85	3.40	3.16	3.61	3.80	4.15	4.27	4.50	4.80	4.90	5.20	5.20	87
2.21	2.58	2.69	3.00	2.92	3.16	3.10	3.30	3.40	3.55	3.59	3.70	3.79	3.88	3.90	4.00	88
2.26	2.50	2.82	3.09	3.18	3.45	3.40	3.71	3.90	4.30	4.28	4.69	4.60	5.10	5.00	5.40	89
1.51	1.82	1.80	2.13	1.95	2.32	2.09	2.48	2.33	2.70	2.50	2.89	2.70	3.08	2.84	3.20	90
2.05	2.35	2.39	2.70	2.58	2.82	2.70	2.97	2.98	3.15	3.16	3.29	3.35	3.40	3.50	3.50	91
1.87	2.55	2.20	3.09	2.40	3.35	2.57	2.59	2.88	4.05	3.09	4.40	3.30	4.75	3.50	5.00	92
2.10	2.60	2.65	3.38	3.00	3.90	3.27	4.31	3.80	5.20	4.19	5.84	4.70	6.70	5.00	7.20	93
2.34	2.42	2.68	2.70	2.80	2.82	2.90	2.91	3.05	3.06	3.12	3.12	3.22	3.22	3.30	3.30	94
2.40	2.65	3.00	3.30	3.40	3.75	3.71	4.10	4.40	4.75	4.99	5.30	5.60	5.85	6.20	6.20	95
2.68	3.45	3.30	4.41	3.65	4.95	3.90	5.39	4.40	6.20	4.78	6.80	5.15	7.50	5.40	8.00	96
1.95	2.20	2.27	2.53	2.46	2.71	2.61	2.86	2.90	3.10	3.11	3.30	3.30	3.45	3.50	3.60	97
2.20	2.46	2.42	2.87	2.55	3.08	2.65	3.26	2.80	3.55	2.91	3.77	3.05	3.97	3.10	4.10	98
3.35	3.50	4.30	4.50	4.95	5.10	5.40	5.58	6.30	6.40	7.00	7.10	7.70	7.90	8.30	8.40	99
2.03	2.45	2.40	2.99	2.60	3.25	2.78	3.50	3.10	3.95	3.32	4.28	3.60	4.65	3.80	5.00	100
2.40	2.95	3.10	3.69	3.52	4.10	3.89	4.45	4.65	5.10	5.22	5.62	5.95	6.30	6.50	6.80	101
2.15	2.52	2.98	3.10	3.45	3.50	3.80	3.80	4.40	4.40	4.90	4.90	5.40	5.40	5.78	5.78	102
2.72	2.82	3.17	3.25	3.35	3.42	3.49	3.53	3.70	3.75	3.80	3.85	3.95	3.95	4 00	4.00	103
2.42	2.70	2.85	3.15	3.08	3.40	3.20	3.60	3.40	3.98	3.58	4.23	3.75	4.55	3.82	4.80	104

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period June 1–September 30 in each year of record.
year of record.

TABLE 3.—

No.	Station	Elevation (feet)	Average annual precipitation through 1939 (inches)	Length of record (years)	RECORD USED FOR FREQUENCY ARRAY		AVERAGE OF MAXIMUM 1-DAY RAIN EACH:	
					Dates (inclusive)	Number years	Summer[1]	Year[2]
							(*a*) NEW MEXICO—	
105	Montoya	4,335	14.06	24	1910–1932	23	1.54	1.77
106	San Jon	3,982	15.69	33	1907–1939	33	1.86	2.18
107	Winsor's Ranch	8,800	24.49	45	1897–1939	43	1.32	1.49
108	Harvey's Upper Ranch	9,400	33.43	15	1909–1923	15	1.80	2.05
109	Onava	6,700	15.78	32	1909–1935; 1937–1939	30	1.47	1.69
110	Gallinas Plant. Station	7,500	25.64	25	1907–1930	24	1.74	1.84
111	Mineral Hill	7,050	21.06	20	1905–1921; 1923	18	1.72	1.84
112	Las Vegas	6,400	17.77	56	1893–1939	47	1.63	1.75
113	Pecos Ranger Station	6,900	15.99	22	1916; 1919–1920; 1923–1939	20	1.29	1.37
114	Doretta	6,381	15.79	21	1915–1920; 1925–1938	20	1.46	1.57
115	Cuervo	4,849	13.26	31	1909–1939	31	1.46	1.75
116	Palma	7,000	11.78	26	1906; 1916–1939	25	1.28	1.42
117	Santa Rosa	4,624	14.28	40	1908–1916; 1919–1939	30	1.38	1.68
118	Pastura	5,285	12.69	31	1909–1928; 1930–1939	30	1.53	1.61
119	Vaughn	5,930	13.01	31	1909–1939	31	1.32	1.58
120	Lagunita	4,500	14.34	22	1905–1913; 1917–1926	19	1.52	1.72
121	Ft. Sumner	4,028	15.55	38	1908–1939	32	1.63	1.96
122	Duran	6,272	14.23	25	1908–1932	25	1.29	1.35
123	Melrose	4,400	16.28	32	1908–1926; 1933–1939	26	1.78	1.99
124	St. Vrain	4,250	16.57	28	1913–1939	27	1.68	1.97
125	Clovis	4,262	18.30	29	1911–1939	29	2.32	2.47
126	Torrance	6,433	12.89	19	1909–1925	17	1.53	1.70
127	Corona	6,666	14.77	31	1909–1939	31	1.29	1.36
128	Portales	4,004	17.55	33	1905–1906; 1912–1939	30	1.92	2.25
129	Gallinas Ranger Station	6,636	14.26	31	1909–1939	31	1.26	1.38
130	Elida	4,345	15.04	27	1913–1939	27	1.75	2.00
131	Ancho	6,112	12.47	30	1909–1939	31	1.35	1.42
132	Richland	4,000	14.51	27	1913–1939	27	1.95	2.04
133	Boaz	4,154	13.16	31	1909–1939	31	1.89	2.07
134	Carrizozo	5,438	13.42	32	1908–1935; 1937–1939	31	1.17	1.25
135	Arabela	6,700	19.54	17	1902–1905; 1916–1921; 1930–1939	20	2.28	2.58
136	Capitan	6,348	17.11	31	1909–1939	31	1.49	1.55
137	Ft. Stanton	6,231	15.96	66	1893; 1897; 1900–1939	42	1.37	1.45
138	Oscura No. 2	5,016	9.95	19	1909–1927	19	.95	1.05
139	Roswell	3,602	14.38	62	1894–1939	46	1.60	1.88
140	Three Rivers	4,436	9.56	24	1910–1932	23	1.01	1.16
141	White Tail	7,000	22.88	56	1915–1918; 1920–1939	24	1.65	1.70
142	Tatum	3,950	15.75	21	1919–1925; 1927; 1929–1939	19	2.07	2.23
143	Mescalero	6,627	19.46	28	1911–1916; 1918–1939	28	1.58	1.65
144	Prairieview	3,825	15.11	28	1911–1916; 1918–1939	28	1.66	1.79
145	Tularosa	4,436	9.32	32	1908–1910; 1914–1939	29	1.40	1.45
146	Cloudcroft	8,650	24.56	38	1903–1939	37	1.61	1.76
147	Elk	5,350	17.42	33	1895–1901; 1904–1909; 1922–1939	31	1.96	2.07
148	Lovington	3,900	14.37	32	1910–1915; 1917–1925; 1927–1928; 1932–1939	25	1.81	1.98
149	Mountain Park	6,720	18.42	25	1912–1924; 1928–1929; 1931–1939	24	1.58	1.79
150	Mayhill Ranger Station	6,400	19.63	23	1917–1939	23	1.89	1.92
151	Alamogordo	4,338	11.18	45	1901–1925; 1927–1939	38	1.23	1.39
152	Artesia	3,350	12.29	33	1905–1906; 1910–1939	32	1.60	1.88
153	Weed Ranger Station	7,200	21.27	15	1905–1906; 1916–1929	16	1.83	1.93
154	Hope	4,000	13.63	21	1906–1908; 1916; 1919–1938	24	1.64	1.86
155	Hobbs	3,600	15.45	22	1913–1929; 1932–1933; 1938–1939	21	2.20	2.24
156	Pearl	3,700	13.02	24	1915–1925; 1927–1928; 1930; 1932–1939	22	1.87	2.06
157	Lakewood	3,325	10.95	17	1912–1925; 1927	15	1.71	2.11
158	Lake Avalon	3,200	11.74	26	1915–1939	25	1.62	1.83
159	Carlsbad	3,120	12.83	48	1894–1899; 1901–1939	45	1.72	1.84
160	Oro Grande	4,171	9.43	33	1905; 1910–1939	31	1.46	1.53
161	Loving	3,045	10.94	22	1918–1939	22	1.27	1.51
162	Carson Seep	6,500	19.29	26	1914–1939	26	2.24	2.42

[1] Average of the maximum 1-day rain recorded for the

[2] Average of the maximum 1-day rain recorded in each

(Continued)

One-Day Rainfall Which Probably Will Be Equaled or Exceeded Once In:																No.
5 years		10 years		15 years		20 years		35 years		50 years		75 years		100 years		
Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	
(Continued)																
2.50	2.70	3.30	3.60	3.80	4.20	4.20	4.70	5.00	5.70	5.60	6.40	6.35	7.30	7.00	8.00	105
2.75	3.25	3.31	4.29	3.65	4.95	3.85	5.40	4.25	6.50	4.55	7.22	4.80	8.20	5.00	9.00	106
1.70	1.90	2.00	2.13	2.16	2.26	2.28	2.35	2.45	2.51	2.59	2.60	2.70	2.70	2.79	2.79	107
2.31	2.66	2.67	3.18	2.85	3.45	3.10	3.69	3.26	4.15	3.44	4.45	3.62	4.80	3.76	5.00	108
2.07	2.36	2.50	2.75	2.75	2.91	2.90	3.01	3.16	3.22	3.30	3.35	3.43	3.46	3.50	3.52	109
2.35	2.45	2.82	2.91	3.12	3.21	3.40	3.43	3.90	3.95	4.30	4.30	4.70	4.70	5.00	5.00	110
2.25	2.33	2.66	2.79	2.91	3.05	3.10	3.25	3.50	3.65	3.72	3.93	4.00	4.25	4.20	4.45	111
2.25	2.25	2.55	2.55	2.75	2.75	2.82	2.83	2.98	3.08	3.05	3.23	3.15	3.40	3.20	3.50	112
1.70	2.10	2.41	2.61	2.74	2.90	2.98	3.10	3.45	3.55	3.80	3.85	4.20	4.20	4.50	4.50	113
2.06	2.18	2.39	2.45	2.50	2.54	2.56	2.60	2.65	2.70	2.70	2.74	2.75	2.79	2.79	2.80	114
2.08	2.50	2.42	3.00	2.60	3.28	2.70	3.45	2.92	3.80	3.06	4.05	3.20	4.30	3.30	4.46	115
1.86	2.06	2.62	2.82	3.15	3.37	3.60	3.80	4.60	4.80	5.40	5.50	6.40	6.40	7.20	7.20	116
1.74	2.18	1.91	2.50	2.03	2.68	2.11	2.80	2.27	3.03	2.35	3.20	2.45	3.35	2.50	3.48	117
2.08	2.12	2.40	2.40	2.56	2.56	2.70	2.70	2.88	2.88	3.00	3.00	3.11	3.11	3.20	3.20	118
1.75	2.10	2.19	2.50	2.45	2.75	2.65	2.95	3.10	3.31	3.40	3.59	3.75	3.86	4.00	4.04	119
2.30	2.30	2.90	2.90	3.22	3.22	3.50	3.50	4.00	4.05	4.30	4.45	4.70	4.95	5.00	5.30	120
2.42	2.80	2.91	3.20	3.16	3.40	3.31	3.52	3.65	3.75	3.85	3.92	4.08	4.10	4.20	4.20	121
1.80	1.81	2.25	2.25	2.55	2.55	2.80	2.80	3.20	3.20	3.50	3.50	3.90	3.90	4.20	4.20	122
2.31	2.58	2.81	3.12	3.13	3.49	3.38	3.80	3.90	4.21	4.25	4.60	4.68	5.00	5.00	5.30	123
2.25	2.40	2.60	2.90	2.80	3.20	2.97	3.41	3.28	3.92	3.50	4.30	3.73	4.65	3.90	4.90	124
3.38	3.50	4.00	4.15	4.30	4.40	4.30	4.60	4.83	4.90	5.01	5.04	5.25	5.30	5.40	5.40	125
2.20	2.20	2.90	2.90	3.35	3.35	3.70	3.70	4.50	4.50	5.10	5.10	5.85	5.85	6.40	6.40	126
1.80	1.90	2.29	2.40	2.60	2.72	2.87	2.99	3.40	3.50	3.80	3.82	4.30	4.30	4.60	4.60	127
2.62	3.00	3.49	3.85	4.05	4.40	4.52	4.88	5.60	5.80	6.35	6.58	7.25	7.40	8.00	8.00	128
1.73	1.83	2.04	2.14	2.21	2.30	2.32	2.40	2.58	2.60	2.71	2.73	2.89	2.89	3.00	3.00	129
2.43	2.75	3.30	3.61	3.88	4.20	4.35	4.67	5.50	5.65	6.20	6.40	7.20	7.30	8.00	8.00	130
1.92	1.92	2.51	2.59	3.04	3.04	3.31	3.38	4.18	4.18	4.70	4.70	5.42	5.42	6.00	6.00	131
2.79	2.79	3.40	3.40	3.60	3.60	3.89	3.89	4.20	4.20	4.50	4.50	4.78	4.78	5.00	5.00	132
2.80	2.80	3.60	3.60	3.81	3.81	4.20	4.20	4.60	4.60	5.00	5.00	5.35	5.35	5.60	5.60	133
1.58	1.65	1.88	1.91	2.05	2.10	2.20	2.21	2.45	2.49	2.65	2.65	2.85	2.85	3.00	3.00	134
3.40	3.65	4.50	4.70	5.20	5.43	5.80	6.00	7.10	7.20	8.00	8.01	9.10	9.10	10.00	10.00	135
2.00	2.00	2.25	2.25	2.32	2.32	2.40	2.40	2.47	2.47	2.51	2.51	2.58	2.58	2.59	2.60	136
1.93	2.05	2.38	2.49	2.62	2.73	2.80	2.90	3.20	3.28	3.48	3.50	3.78	3.80	4.00	4.00	137
1.28	1.37	1.52	1.60	1.68	1.73	1.80	1.81	2.00	2.01	2.13	2.13	2.28	2.28	2.40	2.40	138
2.22	2.63	3.00	3.50	3.55	4.05	4.00	4.50	4.90	5.50	5.60	6.20	6.50	7.10	7.10	7.90	139
1.43	1.62	1.89	2.09	2.18	2.38	2.40	2.60	2.90	3.04	3.25	3.35	3.68	3.72	4.00	4.00	140
2.10	2.15	2.48	2.49	2.69	2.69	2.82	2.82	3.13	3.13	3.34	3.34	3.55	3.55	3.70	3.70	141
2.85	3.23	3.90	4.30	4.67	5.10	5.40	5.70	6.70	6.98	7.70	8.00	9.00	9.10	10.00	10.00	142
2.05	2.08	2.37	2.40	2.56	2.60	2.70	2.72	3.00	3.00	3.18	3.20	3.38	3.39	3.50	3.50	143
2.42	2.42	3.00	3.00	3.16	3.16	3.45	3.45	3.78	3.78	4.08	4.08	4.35	4.35	4.60	4.60	144
2.03	2.04	2.65	2.65	3.05	3.05	3.40	3.40	4.13	4.13	4.65	4.65	5.30	5.30	5.80	5.80	145
2.10	2.32	2.45	2.65	2.65	2.82	2.80	2.95	3.05	3.15	3.20	3.25	3.40	3.40	3.50	3.50	146
2.75	2.90	3.50	3.60	3.95	4.05	4.30	4.40	5.05	5.10	5.55	5.60	6.20	6.20	6.60	6.60	147
2.60	2.85	3.25	3.50	3.68	3.90	4.00	4.20	4.62	4.78	5.10	5.20	5.60	5.63	6.00	6.00	148
2.28	2.45	2.88	2.98	3.20	3.28	3.45	3.50	3.95	4.00	4.30	4.30	4.70	4.70	5.00	5.00	149
2.50	2.55	2.90	2.90	3.08	3.10	3.20	3.20	3.42	3.42	3.55	3.55	3.69	3.69	3.80	3.80	150
1.64	1.78	2.12	2.25	2.45	2.58	2.70	2.81	3.25	3.35	3.65	3.74	4.15	4.20	4.50	4.50	151
2.10	2.50	2.71	3.18	3.15	3.64	3.50	4.00	4.32	4.75	4.98	5.30	5.78	5.95	6.40	6.40	152
2.43	2.49	2.75	2.75	2.89	2.90	3.00	3.00	3.18	3.20	3.30	3.30	3.45	3.45	3.50	3.50	153
2.35	2.42	3.05	3.05	3.47	3.47	3.80	3.80	4.50	4.50	5.00	5.00	5.55	5.55	6.00	6.00	154
3.38	3.48	4.20	4.30	4.65	4.75	5.00	5.05	5.70	5.75	6.10	6.20	6.60	6.70	7.00	7.00	155
2.80	2.80	3.40	3.40	3.70	3.70	4.00	4.00	4.45	4.45	4.80	4.80	5.10	5.10	5.40	5.40	156
2.62	3.23	3.50	4.10	4.05	4.64	4.50	5.00	5.40	5.80	6.05	6.40	6.80	7.00	7.50	7.50	157
2.40	2.50	2.81	2.87	3.05	3.10	3.21	3.25	3.54	3.55	3.71	3.71	3.95	3.95	4.10	4.10	158
2.46	2.67	3.15	3.40	3.60	3.82	3.90	4.15	4.60	4.81	5.10	5.30	5.70	5.80	6.20	6.20	159
2.10	2.10	2.61	2.61	2.82	2.82	3.15	3.15	3.50	3.50	3.83	3.83	4.15	4.15	4.40	4.40	160
1.83	2.03	2.30	2.40	2.55	2.69	2.80	2.90	3.18	3.28	3.50	3.50	3.85	3.85	4.10	4.10	161
3.30	3.45	4.30	4.40	4.85	4.93	5.30	5.38	6.15	6.20	6.79	6.80	7.45	7.45	8.00	8.00	162

period June 1–September 30 in each year of record.
year of record.

TABLE 3.—

No.	Station	Elevation (feet)	Average annual precipitation through 1938 (inches)	Length of record (years)	RECORD USED FOR FREQUENCY ARRAY		AVERAGE OF MAXIMUM 1-DAY RAIN EACH:	
					Dates (inclusive)	Number years	Summer[1]	Year[2]
								(*b*) ARI-
163	San Carlos Reservoir...	2,532	12.12	50[3]	1885–1887; 1898–1901; 1903–1911; 1917–1918; 1921–1923	26	1.20	1.47
164	Thatcher.............	2,800	9.55	39	1904–1937	34	1.19	1.32
165	Clifton...............	3,465	13.43	50	1906–1907; 1909–1940	34	1.31	1.43
166	Benson...............	3,523	11.07	58	1898–1940	43	1.18	1.24
167	San Simon...........	3,609	7.68	48[3]	1899–1900; 1903–1910; 1913–1914; 1922–1923; 1925–1940	30	1.07	1.24
168	Bowie...............	3,756	10.77	66	1881–1887; 1899–1901; 1903–1916; 1919–1939	45	1.17	1.36
169	Fairbank............	3,862	12.00	17	1910–1918; 1924–1940	26	1.41	1.48
170	Douglas.............	3,939	12.78	35	1904–1940	37	1.30	1.43
171	Lewis Springs.........	4,029	11.17	22	1910–1918; 1920–1933	23	1.45	1.49
172	Allaires Ranch........	4,184	11.55	21	1895–1913	19	1.50	1.61
173	Willcox..............	4,200	11.99	57[3]	1888–1890; 1898–1921; 1923–1925; 1927–1938; 1940	43	1.30	1.46
174	Cochise..............	4,250	11.39	27[3]	1899; 1901–1923	24	1.22	1.44
175	Leslie Canyon.........	4,461	13.44	23	1916–1918; 1920–1940	24	1.44	1.48
176	Naco................	4,579	14.44	25	1914–1918; 1925–1934	15	1.62	1.79
177	Tombstone...........	4,540	14.85	41	1897–1899; 1901–1940	43	1.51	1.60
178	Ft. Grant............	4,833	13.32	65	1878–1891; 1898–1905: 1918–1919; 1921–1923; 1926–1928; 1931; 1933–1940	39	1.36	1.58
179	Ft. Huachuca.........	5,100	16.96	35	1900–1918	19	1.44	1.75
180	Ft. Apache...........	5,300	18.45	62	1879–1888; 1897–1918; 1921–1933	45	1.36	1.67
181	Bisbee...............	5,425	19.98	50	1893–1929; 1931–1940	47	1.79	1.99
182	Paradise.............	5,436	18.73	32	1914–1937	24	1.43	1.87
183	Rucker Canyon........	5,634	21.38	28	1917; 1924–1940	18	1.73	1.89
184	Snowflake............	5,644	12.98	39	1897–1900; 1911–1938; 1940	33	1.21	1.27
185	Portal...............	6,000	18.53	25	1914–1917; 1921–1923; 1926–1937; 1940	20	1.69	1.86
186	Show Low...........	6,300	17.64	30[3]	1893–1898; 1909; 1916–1925	17	1.14	1.63
187	Heber...............	6,484	18.94	15[3]	1915–1921; 1926–1930; 1935	13	1.35	1.98
188	Blue.................	6,500	18.89	27	1914–1930	15	1.30	1.59
189	Pinedale.............	6,500	19.07	26	1912–1940	29	1.38	1.68
190	Springerville..........	6,862	11.79	40	1911–1940	30	1.12	1.17
191	Henrys Camp.........	6,925	23.11	23	1916–1938	23	1.28	1.67
192	Lakeside.............	7,054	22.91	20[3]	1911–1926; 1929–1933	21	1.43	1.93
193	Alpine...............	8,500	23.28	29[3]	1905; 1912–1917; 1920–1922; 1925–1933; 1935–1940	25	1.46	1.76
194	Greer................	8,500	22.18	22[3]	1904–1910; 1913–1916; 1918; 1920–1923; 1925–1926	18	1.23	1.42
195	Buckeye.............	980	7.80	50	1893–1940	48	1.03	1.37
196	Phoenix W. B.........	1,108	7.78	62	1897–1940	44	1.10	1.27
197	Marinette............	1,150	7.82	25[3]	1913–1915; 1917–1918; 1921–1923; 1925–1940	24	.97	1.40
198	Tempe Date Orchard...	1,165	8.74	38	1905–1940	36	1.04	1.33
199	Litchfield Park........	1,180	8.22	21	1918–1937; 1939–1940	22	1.17	1.45
200	Maricopa............	1,186	8.12	64	1898–1940	43	1.26	1.41
201	Goulds Ranch.........	1,195	7.41	24	1915–1940	26	.97	1.35
202	Goodyear............	1,203	7.78	14	1919–1932	15	1.01	1.24
203	Chandler............	1,213	8.16	22	1913–1933	21	1.10	1.38
204	Mesa Experimental Farm	1,245	8.69	44	1896–1912; 1915–1940	43	1.02	1.26
205	Sacaton..............	1,280	9.60	31	1908–1940	33	1.28	1.44
206	Granite Reef Dam.....	1,325	9.52	46	1893–1894; 1896–1923; 1925–1940	46	.97	1.31
207	Casa Grande..........	1,400	8.30	50[3]	1898–1908; 1910–1914; 1917–1919; 1921–1922; 1925–1938; 1940	36	1.14	1.36
208	Casa Grande Ruin.....	1,422	9.73	23[3]	1906; 1909–1915; 1932–1937; 1940	15	1.25	1.62
209	Florence.............	1,500	9.88	38[3]	1893; 1907; 1910; 1912–1917; 1921; 1923–1940	28	1.18	1.37
210	Morman Flats.........	1,815	13.54	14[3]	1923–1937	14	1.64	1.87
211	Redrock.............	1,856	9.61	35[3]	1893–1894; 1908–1933; 1935	29	1.17	1.38
212	Canon...............	1,990	17.92	15[3]	1916–1929	12	1.57	2.34
213	Dudleyville...........	2,204	14.60	37	1893–1909; 1911–1923	30	1.34	1.56
214	Intake...............	2,230	13.41	33	1907–1940	34	1.29	1.55
215	Roosevelt...........	2,275	17.86	34	1905–1910; 1912–1940	35	1.09	1.60
216	Gisela...............	2,300	18.78	27[3]	1916–1923; 1929–1940	20	1.19	1.57

[1] Average of the maximum 1-day rain recorded for the

[2] Average of the maximum 1-day rain recorded in each

[3] Some missing months interpolated from surrounding

(Continued)

One-Day Rainfall Which Probably Will Be Equaled or Exceeded Once In:																
5 years		10 years		15 years		20 years		35 years		50 years		75 years		100 years		No.
Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	
ZONA																
1.75	1.88	2.21	2.40	2.49	2.65	2.70	2.81	3.09	3.19	3.37	3.41	3.70	3.72	3.91	3.91	163
1.65	1.75	1.98	2.00	2.15	2.15	2.25	2.26	2.45	2.45	2.55	2.57	2.70	2.70	2.79	2.80	164
1.78	1.81	1.92	1.93	1.98	1.98	2.00	2.01	2.05	2.05	2.08	2.08	2.09	2.09	2.09	2.10	165
1.54	1.60	1.82	1.88	1.98	2.03	2.09	2.12	2.30	2.33	2.42	2.47	2.58	2.60	2.70	2.70	166
1.42	1.58	1.71	2.08	1.88	2.38	2.00	2.63	2.22	3.16	2.40	3.55	2.61	4.00	2.75	4.35	167
1.59	1.76	1.79	2.00	1.90	2.07	1.92	2.18	2.05	2.24	2.09	2.30	2.16	2.36	2.17	2.40	168
2.09	2.10	2.48	2.49	2.62	2.62	2.70	2.70	2.82	2.82	2.90	2.90	2.95	2.95	3.00	3.00	169
1.65	1.84	2.09	2.28	2.35	2.54	2.58	2.72	3.00	3.15	3.30	3.40	3.69	3.75	4.00	4.00	170
1.79	1.84	2.20	2.25	2.46	2.51	2.68	2.71	3.08	3.12	3.36	3.41	3.70	3.78	3.98	4.01	171
2.06	2.18	2.42	2.49	2.60	2.63	2.70	2.72	2.90	2.91	3.01	3.01	3.15	3.15	3.22	3.22	172
1.70	2.00	2.10	2.44	2.40	2.69	2.60	2.89	3.04	3.25	3.35	3.50	3.72	3.79	4.00	4.00	173
1.78	1.93	2.18	2.29	2.35	2.45	2.50	2.58	2.73	2.80	2.90	2.95	3.08	3.10	3.20	3.20	174
1.97	2.00	2.40	2.40	2.62	2.62	2.80	2.80	3.12	3.12	3.39	3.39	3.59	3.59	3.79	3.79	175
2.32	2.49	2.91	3.02	3.25	3.34	3.49	3.58	3.97	4.01	4.30	4.30	4.68	4.68	5.00	5.00	176
1.96	2.15	2.31	2.50	2.50	2.68	2.61	2.79	2.85	2.98	3.00	3.10	3.18	3.21	3.30	3.30	177
1.96	2.13	2.40	2.45	2.60	2.63	2.75	2.79	3.00	3.00	3.20	3.20	3.40	3.40	3.55	3.55	178
1.76	2.16	1.92	2.39	2.00	2.50	2.02	2.60	2.10	2.72	2.18	2.81	2.18	2.90	2.20	2.98	179
1.79	2.15	2.21	2.52	2.49	2.75	2.70	2.94	3.11	3.29	3.40	3.50	3.77	3.81	4.00	4.00	180
2.32	2.55	2.82	3.10	3.16	3.40	3.40	3.67	3.91	4.15	4.30	4.50	4.70	4.90	5.00	5.20	181
1.99	2.40	2.32	3.00	2.48	3.30	2.58	3.54	2.75	4.00	2.87	4.33	2.98	4.74	3.05	5.00	182
2.10	2.27	2.35	2.50	2.48	2.65	2.58	2.75	2.71	2.91	2.81	3.00	2.92	3.14	3.00	3.20	183
1.60	1.60	1.95	1.95	2.15	2.15	2.30	2.31	2.62	2.65	2.85	2.89	3.10	3.15	3.30	3.30	184
2.02	2.26	2.20	2.48	2.28	2.69	2.34	2.64	2.45	2.79	2.50	2.85	2.60	2.95	2.65	3.00	185
1.33	2.06	1.52	2.30	1.65	2.43	1.72	2.51	1.90	2.70	2.00	2.80	2.11	2.90	2.20	3.00	186
1.83	2.60	2.18	3.18	2.34	3.36	2.48	3.50	2.70	3.69	2.89	3.80	3.06	3.94	3.20	4.00	187
1.83	2.14	2.06	2.39	2.18	2.50	2.25	2.59	2.38	2.70	2.48	2.79	2.52	2.83	2.60	2.90	188
1.65	2.04	1.86	2.35	1.97	2.50	2.06	2.59	2.22	2.73	2.30	2.81	2.43	2.92	2.50	3.00	189
1.42	1.46	1.69	1.70	1.82	1.84	1.92	1.92	2.12	2.12	2.28	2.28	2.40	2.40	2.50	2.50	190
1.65	2.18	2.02	2.71	2.28	3.08	2.48	3.35	2.85	3.95	3.12	4.35	3.43	4.85	3.65	5.20	191
1.76	2.38	2.20	2.80	2.45	3.04	2.65	3.19	3.07	3.46	3.37	3.65	3.70	3.85	4.00	4.00	192
1.81	2.18	2.18	2.45	2.40	2.58	2.52	2.64	2.75	2.79	2.85	2.89	2.95	2.95	3.00	3.00	193
1.60	1.84	1.87	2.10	1.97	2.25	2.05	2.32	2.19	2.49	2.28	2.59	2.35	2.69	2.40	2.74	194
1.57	1.79	2.15	2.20	2.48	2.49	2.70	2.70	3.09	3.09	3.40	3.40	3.75	3.75	4.00	4.00	195
1.59	1.78	2.16	2.16	2.40	2.40	2.50	2.50	2.80	2.80	3.00	3.00	3.08	3.08	3.20	3.20	196
1.50	1.95	2.10	2.50	2.46	2.84	2.76	3.10	3.40	3.69	3.89	4.10	4.45	4.60	4.90	5.00	197
1.50	1.66	1.91	2.01	2.17	2.25	2.35	2.40	2.72	2.76	3.00	3.00	3.29	3.29	3.50	3.50	198
1.96	2.22	2.39	2.61	2.56	2.75	2.66	2.82	2.81	2.91	2.90	2.95	2.98	2.98	3.00	3.00	199
1.98	2.06	2.62	2.69	2.95	2.98	3.10	3.14	3.39	3.40	3.50	3.52	3.62	3.65	3.70	3.70	200
1.55	1.90	2.02	2.15	2.21	2.25	2.30	2.30	2.40	2.40	2.45	2.45	2.48	2.48	2.50	2.50	201
1.62	1.77	1.99	2.03	2.17	2.20	2.31	2.31	2.54	2.54	2.70	2.70	2.85	2.85	3.00	3.00	202
1.69	1.92	2.15	2.29	2.39	2.49	2.55	2.63	2.88	2.93	3.09	3.11	3.31	3.33	3.50	3.50	203
1.43	1.62	1.85	2.00	2.08	2.22	2.30	2.39	2.67	2.74	2.91	2.99	3.25	3.28	3.50	3.50	204
1.83	2.01	2.18	2.27	2.33	2.39	2.44	2.50	2.65	2.68	2.79	2.79	2.90	2.90	3.00	3.00	205
1.64	1.87	1.90	2.04	1.95	2.13	2.01	2.20	2.05	2.31	2.08	2.39	2.09	2.48	2.10	2.50	206
1.63	1.87	2.30	2.51	2.73	2.96	3.11	3.30	3.95	4.10	4.58	4.70	5.40	5.40	6.00	6.00	207
1.71	1.90	2.28	2.38	2.60	2.68	2.81	2.90	3.35	3.40	3.70	3.80	4.13	4.20	4.50	4.50	208
1.72	1.76	1.93	1.93	1.98	1.98	2.00	2.00	2.05	2.05	2.08	2.08	2.08	2.08	2.09	2.09	209
2.28	2.61	2.81	3.20	3.18	3.50	3.41	3.70	3.91	4.15	4.29	4.45	4.68	4.75	5.00	5.00	210
1.76	1.89	1.96	2.06	2.03	2.12	2.08	2.16	2.18	2.21	2.21	2.23	2.28	2.28	2.30	2.30	211
2.44	3.40	2.90	4.00	3.10	4.27	3.22	4.50	3.45	4.79	3.60	5.00	3.71	5.20	3.80	5.40	212
1.89	2.02	2.22	2.30	2.38	2.44	2.49	2.52	2.68	2.70	2.79	2.80	2.90	2.90	3.00	3.00	213
1.86	2.13	2.21	2.52	2.38	2.72	2.49	2.87	2.68	3.10	2.80	3.21	2.92	3.40	3.00	3.50	214
1.47	2.04	1.70	2.30	1.83	2.44	1.91	2.53	2.06	2.70	2.18	2.80	2.28	2.92	2.32	3.00	215
1.62	1.88	1.85	2.04	1.98	2.13	2.05	2.20	2.20	2.31	2.30	2.38	2.41	2.45	2.49	2.50	216

period June 1–September 30 in each year of record.
year of record.
stations for computation of average annual precipitation.

TABLE 3.—

No.	Station	Elevation (feet)	Average annual precipitation through 1938 (inches)	Length of record (years)	RECORD USED FOR FREQUENCY ARRAY		AVERAGE OF MAXIMUM 1-DAY RAIN EACH:	
					Dates (inclusive)	Number years	Summer[1]	Year[2]
							(b) ARIZONA—	
217	Tucson, Univ. of Ariz.	2,423	11.28	72	1915–1940	26	1.32	1.46
218	Childs (Verde Hot Springs)	2,650	16.93	24	1915–1940	26	1.38	2.17
219	Superior	2,990	18.63	19	1920–1940	21	1.52	1.81
220	Alamo Ranger Station	3,155	13.88	24[3]	1916–1940	25	1.34	1.48
221	Vail	3,241	9.07	14[3]	1899–1910; 1912–1915	16	1.08	1.26
222	Globe	3,440	16.17	20	1894; 1901–1902; 1905; 1907–1928; 1930–1940	37	1.29	1.50
223	Clemenceau	3,465	11.31	17	1920–1936	17	1.21	1.38
224	Walnut Grove	3,469	17.79	49	1897–1940	45	1.51	2.28
225	Miami	3,603	19.82	25	1914–1940	27	1.47	1.83
226	Silverbell	3,604	13.19	15	1906–1915; 1917–1920	14	1.32	1.57
227	Cordes	3,773	12.89	14	1926–1940	15	1.04	1.55
228	Nogales	3,839	17.19	33	1900–1903; 1909; 1914–1940	32	1.51	1.66
229	Fresnal Ranch	4,000	18.40	15[3]	1923–1929; 1931–1937	14	1.43	1.77
230	Santa Marguerita	4,000	19.28	22	1917–1940	24	1.82	1.98
231	Sycamore Ranger Station	4,000	21.89	20[3]	1919–1921; 1924–1940	20	1.62	2.01
232	Patagonia	4,044	17.85	18	1922–1940	19	1.46	1.52
233	Helvetia	4,300	19.70	24[3]	1916–1919; 1921–1940	24	1.69	1.81
234	Young	4,400	21.75	29	1918; 1920–1923; 1925–1927; 1929–1935; 1938–1940	18	1.46	1.97
235	Pinal Ranch	4,520	25.57	46	1895–1940	46	1.65	2.44
236	Oracle	4,522	19.72	46[3]	1894–1910; 1913–1921; 1923–1927; 1929–1940	43	1.44	1.93
237	Tonto Ranger Station	4,732	16.36	24	1915–1928; 1930–1937; 1940	23	1.53	2.04
238	Rosemont	4,800	17.73	17	1914–1918; 1923–1930	13	1.42	1.87
239	Yeager Canyon	4,850	18.10	22	1918–1940	23	1.45	2.14
240	Elgin	4,900	15.85	27	1913–1940	28	1.62	1.73
241	Payson	4,906	21.04	40[3]	1893; 1895–1897; 1909–1916; 1918–1940	35	1.56	2.17
242	Natural Bridge	4,607	25.50	49	1893–1940	48	1.62	2.25
243	Canille	5,225	19.82	28	1910–1921; 1923–1940	30	1.89	1.98
244	Jerome	5,250	19.12	42	1898–1940	43	1.32	1.83
245	Prescott	5,354	20.52	73	1916–1940	25	1.31	2.17
246	Crown King	6,000	28.09	24	1915–1918; 1920–1940	25	1.96	3.39
247	Yuma	141	3.47	69	1898–1940	43	.90	1.10
248	Parker	350	5.16	46	1894–1898; 1900–1905; 1907–1940	45	.80	1.24
249	Aztec	492	4.81	23	1899–1921	23	.68	1.12
250	Mohawk	538	4.12	56	1900–1940	41	.83	1.10
251	Sentinel	685	4.57	21[3]	1899–1918	20	.90	1.13
252	Gila Bend	737	5.88	45[3]	1900–1925; 1928–1940	39	.81	1.05
253	Quartzsite	871	6.19	24	1908–1910; 1912–1914; 1920–1940	27	.95	1.24
254	Signal	1,652	7.33	18[3]	1893–1906	13	.83	1.26
255	Ajo	1,770	9.16	26	1914–1940	27	1.12	1.44
256	Salome	1,775	8.22	30	1908–1912; 1914–1940	32	1.01	1.44
257	Wickenburg	2,072	9.55	38[3]	1908; 1910–1940	32	1.25	1.64
258	Congress	3,688	13.55	14	1897–1910	14	1.22	1.76
259	Yarnell	4,782	19.10	25[3]	1899–1910; 1925–1926; 1936–1939	18	1.56	2.13
260	Leupp	4,150	7.07	12	1915–1921; 1925; 1934–1940	15	.69	.76
261	Tuba City	4,900	6.29	44	1897; 1899–1902; 1904; 1906; 1908–1917; 1920–1922; 1924–1927; 1930–1940	35	.75	.89
262	Winslow	4,848	8.60	40	1899–1900; 1909–1910; 1915–1940	30	1.00	1.08
263	Holbrook	5,069	9.25	42	1893–1899; 1904–1940	44	.97	1.11
264	St. Johns	5,650	12.17	39[3]	1902; 1904; 1906–1907; 1909–1918; 1920–1940	35	1.06	1.11
265	Pinto	5,660	11.83	19	1905–1922	18	1.06	1.20
266	Kayenta	5,800	8.80	26	1915–1917; 1919–1938; 1940	24	.82	1.09
267	Chin Lee	6,090	8.75	25	1909–1924; 1927; 1936–1940	22	.92	.96
268	Ft. Defiance	6,950	13.68	30	1897–1905; 1930–1940	20	.94	1.16
269	St. Michaels	6,950	13.15	22	1906–1925	20	1.12	1.24
270	Ft. Mohave	540	5.09	58	1893; 1896–1910; 1912–1913; 1926–1928; 1930–1934; 1916–1919; 1922–1937	26	.81	1.04
271	Lee's Ferry	3,142	6.20	22	1916–1919; 1922–1937	20	.73	.82

[1] Average of the maximum 1-day rain recorded for the

[2] Average of the maximum 1-day rain recorded in each

[3] Some missing months interpolated from surrounding

(Continued)

One-Day Rainfall Which Probably Will Be Equaled or Exceeded Once In:																No.
5 years		10 years		15 years		20 years		35 years		50 years		75 years		100 years		
Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	
(Continued)																
1.85	2.12	2.30	2.51	2.50	2.72	2.68	2.85	3.00	3.10	3.20	3.25	3.43	3.43	3.60	3.60	217
1.73	2.57	2.38	3.30	2.80	3.76	3.19	4.20	4.00	5.00	4.64	5.60	5.42	6.40	6.00	7.00	218
2.19	2.38	2.61	2.72	2.85	2.95	3.00	3.10	3.35	3.40	3.56	3.60	3.80	3.82	4.00	4.00	219
1.92	2.03	2.30	2.37	2.50	2.55	2.61	2.66	2.87	2.90	3.00	3.05	3.18	3.20	3.30	3.30	220
1.75	1.97	2.10	2.31	2.28	2.48	2.38	2.52	2.49	2.61	2.57	2.65	2.65	2.68	2.70	2.70	221
1.61	1.98	1.90	2.28	2.08	2.41	2.21	2.51	2.49	2.70	2.66	2.80	2.86	2.91	3.00	3.00	222
1.78	1.93	2.68	2.81	3.35	3.50	3.90	4.08	5.30	5.40	6.40	6.50	7.85	7.90	9.00	9.00	223
2.02	2.96	2.58	3.70	2.95	4.23	3.20	4.60	3.75	5.40	4.15	6.00	4.61	6.70	5.00	7.10	224
1.91	2.25	2.28	2.60	2.48	2.81	2.61	2.99	2.92	3.25	3.11	3.45	3.35	3.65	3.50	3.80	225
2.00	2.05	2.19	2.20	2.25	2.26	2.30	2.30	2.38	2.38	2.40	2.40	2.45	2.45	2.50	2.50	226
1.61	2.35	2.28	3.00	2.72	3.44	3.10	3.79	3.94	4.45	4.57	4.90	5.40	5.50	6.00	6.00	227
1.91	2.03	2.30	2.50	2.55	2.79	2.71	3.00	3.09	3.48	3.32	3.80	3.59	4.19	3.80	4.45	228
1.88	2.10	1.99	2.26	2.03	2.31	2.07	2.36	2.12	2.41	2.16	2.45	2.19	2.47	2.20	2.50	229
2.48	2.60	3.20	3.30	3.63	3.75	4.00	4.10	4.74	4.85	5.30	5.40	5.95	6.00	6.40	6.50	230
2.55	2.75	3.08	3.15	3.30	3.34	3.42	3.45	3.66	3.66	3.80	3.80	3.90	3.90	4.00	4.00	231
1.88	2.03	2.29	2.45	2.55	2.70	2.71	2.89	3.12	3.25	3.40	3.50	3.75	3.77	4.00	4.00	232
2.28	2.40	2.60	2.63	2.75	2.75	2.81	2.82	2.95	2.98	3.08	3.08	3.15	3.15	3.20	3.20	233
2.11	2.71	2.51	3.05	2.70	3.19	2.82	3.25	3.06	3.36	3.20	3.40	3.38	3.45	3.50	3.50	234
2.28	3.10	2.69	3.80	2.90	4.27	3.04	4.60	3.30	5.30	3.49	5.79	3.65	6.35	3.80	6.80	235
1.98	2.50	2.30	3.00	2.43	3.31	2.51	3.55	2.70	4.00	2.80	4.30	2.91	4.70	3.00	5.00	236
2.11	2.76	2.63	3.18	2.93	3.40	3.15	3.57	3.59	3.88	3.89	4.10	4.23	4.31	4.50	4.50	237
1.89	2.40	2.22	2.95	2.43	3.28	2.60	3.50	2.90	4.00	3.10	4.30	3.32	4.58	3.50	5.00	238
1.96	2.55	2.35	3.00	2.55	3.23	2.70	3.40	3.00	3.80	3.20	4.00	3.40	4.30	3.50	4.50	239
2.11	2.11	2.50	2.55	2.70	2.78	2.82	2.96	3.08	3.30	3.21	3.55	3.40	3.81	3.50	4.00	240
2.18	2.70	2.58	3.41	2.80	3.88	2.99	4.22	3.32	4.95	3.55	5.50	3.80	6.15	4.00	6.60	241
2.06	2.90	2.72	3.51	3.19	3.90	3.56	4.20	4.33	4.80	4.91	5.22	5.62	5.75	6.20	6.20	242
2.50	2.60	3.12	3.23	3.50	3.60	3.80	3.91	4.40	4.48	4.80	4.90	5.30	5.35	5.60	5.70	243
1.83	2.41	2.11	2.85	2.25	3.08	2.31	3.20	2.46	3.50	2.53	3.70	2.62	3.89	2.68	4.00	244
1.70	2.87	1.88	3.48	1.96	3.82	2.01	4.10	2.12	4.68	2.20	5.01	2.25	5.50	2.30	5.80	245
2.60	4.70	3.49	5.98	4.05	6.65	5.54	7.20	5.60	8.40	6.40	9.20	7.35	10.01	8.00	10.80	246
1.30	1.57	1.92	2.18	2.32	2.52	2.62	2.82	3.32	3.46	3.83	3.95	4.49	4.51	5.00	5.00	247
1.05	1.60	1.46	1.98	1.72	2.20	1.91	2.39	2.38	2.75	2.70	3.00	3.12	3.30	3.45	3.51	248
.96	1.62	1.12	1.88	1.20	1.98	1.25	2.03	1.31	2.14	1.34	2.20	1.37	2.26	1.40	2.30	249
1.27	1.56	1.68	1.91	1.86	2.10	1.99	2.22	2.18	2.50	2.30	2.68	2.42	2.87	2.50	3.00	250
1.43	1.53	1.90	1.91	2.15	2.15	2.31	2.31	2.72	2.72	3.00	3.00	3.28	3.28	3.50	3.50	251
1.17	1.47	1.37	1.71	1.45	1.82	1.50	1.90	1.58	2.05	1.61	2.14	1.67	2.24	1.70	2.30	252
1.35	1.69	1.90	2.21	2.28	2.59	2.60	2.89	3.30	3.48	3.81	3.95	4.50	4.50	5.00	5.00	253
1.06	1.64	1.28	1.81	1.38	1.90	1.45	1.92	1.58	2.02	1.67	2.08	1.73	2.09	1.80	2.10	254
1.53	2.10	1.71	2.52	2.14	2.75	2.32	2.91	2.69	3.22	2.95	3.40	3.26	3.65	3.50	3.80	255
1.47	2.05	1.91	2.41	2.18	2.60	2.33	2.70	2.65	2.90	2.85	3.01	3.05	3.16	3.20	3.25	256
1.87	2.03	2.30	2.39	2.53	2.58	2.70	2.72	2.99	3.02	3.19	3.20	3.40	3.43	3.55	3.60	257
2.02	2.64	2.60	2.85	2.75	2.91	2.80	3.00	2.90	3.04	2.95	3.09	2.99	3.09	3.00	3.10	258
2.00	2.81	2.29	3.21	2.41	3.40	2.51	3.50	2.70	3.71	2.80	3.81	2.91	3.95	3.00	4.00	259
.98	1.03	1.15	1.18	1.20	1.22	1.23	1.27	1.30	1.31	1.34	1.36	1.38	1.38	1.40	1.41	260
1.05	1.19	1.32	1.48	1.49	1.65	1.60	1.78	1.84	2.03	2.00	2.20	2.19	2.40	2.31	2.54	261
1.38	1.48	1.73	1.81	1.95	2.01	2.09	2.15	2.40	2.43	2.60	2.61	2.85	2.85	3.00	3.00	262
1.31	1.40	1.65	1.71	1.83	1.90	1.99	2.05	2.30	2.35	2.51	2.58	2.78	2.82	2.98	3.00	263
1.33	1.42	1.63	1.70	1.80	1.88	1.95	2.00	2.23	2.26	2.42	2.42	2.65	2.65	2.80	2.80	264
1.34	1.52	1.61	1.81	1.80	2.01	1.94	2.15	2.26	2.42	2.49	2.61	2.75	2.85	2.97	3.00	265
1.30	1.50	1.51	1.93	1.55	2.22	1.58	2.47	1.60	2.93	1.61	3.35	1.65	3.65	1.66	4.00	266
1.29	1.29	1.52	1.55	1.67	1.67	1.70	1.72	1.83	1.83	1.89	1.90	1.96	1.96	2.00	2.00	267
1.16	1.46	1.26	1.62	1.31	1.74	1.34	1.80	1.40	1.95	1.42	2.03	1.47	2.12	1.50	2.20	268
1.45	1.60	1.55	1.92	1.59	2.10	1.61	2.24	1.66	2.49	1.69	2.66	1.71	2.85	1.71	3.00	269
1.32	1.65	1.90	1.98	2.05	2.10	2.10	2.20	2.19	2.31	2.22	2.40	2.26	2.48	2.29	2.51	270
1.03	1.25	1.39	1.60	1.60	1.80	1.78	1.98	2.15	2.30	2.42	2.53	2.77	2.81	3.00	3.00	271

period June 1–September 30 in each year of record.
year of record.
stations for computation of average annual precipitation.

TABLE 3.—

No.	Station	Elevation (feet)	Average annual precipitation through 1938 (inches)	Length of record (years)	Record Used for Frequency Array		Average of Maximum 1-Day Rain Each:	
					Dates (inclusive)	Number years	Summer[1]	Year[2]
							(*b*) Arizona—	
272	Supai	3,200	8.89	25	1914–1915; 1917–1918; 1921–1922; 1924–1927; 1930–1932	13	.89	1.17
273	Kingman	3,326	10.92	40	1901–1902; 1904–1940	39	1.13	1.74
274	Truxton	3,997	9.91	21	1910–1917; 1921–1922; 1924–1925; 1927–1940	26	1.24	1.46
275	Cedar Glade	4,610	14.30	24	1915–1940	26	1.28	1.68
276	Prescott Dry Farm	5,008	14.34	18	1913–1928	16	1.47	1.59
277	Walnut Creek Ranger Station	5,138	19.15	24	1916–1940	25	1.42	1.95
278	Ashfork	5,160	12.60	30	1909–1910; 1912–1913; 1915–1928; 1930–1940	29	1.20	1.44
279	Seligman	5,219	11.04	34[3]	1905–1909; 1912–1935; 1937–1940	33	1.14	1.37
280	Williams	6,750	22.32	43	1904–1905; 1908–1940	35	1.39	1.97
281	Grand Canyon	6,866	17.22	38	1904; 1906–1940	36	1.10	1.38
282	Flagstaff	6,907	22.80	51	1894; 1897–1940	45	1.20	1.62
283	Ft. Valley	7,500	24.33	30	1914–1924; 1935–1940	17	1.25	1.73

[1] Average of the maximum 1-day rain recorded for the

[2] Average of the maximum 1-day rain recorded in each

[3] Some missing months interpolated from surrounding

given year, in which case it was necessary to decide whether to include the year. The decision was made after inspection of the complete record to determine what quantity constituted a high rainfall at that station. Depending on the number and season of the missing months, the year was included in the array if it appeared probable that the available record included the day of highest rainfall for the year.

If the data do not plat as a straight line in the recurrence-interval curve, the relatively few widely-spaced points in the upper limits of the series determine how the curve will be drawn and will control the extrapolation. In this study two curves (summer and all-year) were drawn for each of 283 stations. They represent stations in all kinds of topography in regions of average annual precipitation varying from 8 to 30 in. By far the most important factor affecting the extrapolation of the curves was neither the manner of choosing the series, nor the length of record, but the occurrence of aberrant points in the upper limits of the series. When such an aberrancy occurred the position of the curve was mostly a matter of judgment.

In the Southwestern States there is great variability in both the average annual precipitation and in the magnitude of the highest one-day rainfalls that occur in successive years. One year may include a number of heavy rainfalls and may have a high total precipitation, whereas the following year may be dry, with no high rains. To demonstrate this variability the heaviest rain of record each year was used in the present analysis. The recurrence-interval array, therefore, is comparable to the method used by the late Allen Hazen,[6] M. Am. Soc. C. E., in which he computed from stream-flow records

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[6] "Flood Flows," by Allen Hazen, John Wiley and Sons, New York, N. Y., 1930.

(*Continued*)

One-Day Rainfall Which Probably Will Be Equaled or Exceeded Once In:																No.
5 years		10 years		15 years		20 years		35 years		50 years		75 years		100 years		
Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	
(*Continued*)																
1.28	1.95	1.65	2.60	1.82	2.90	1.93	3.11	2.14	3.47	2.29	3.68	2.40	3.86	2.50	4.00	272
1.83	2.52	2.31	3.10	2.61	3.40	2.71	3.61	2.89	4.05	2.95	4.31	2.99	4.65	3.00	4.90	273
1.83	2.24	2.51	2.82	2.90	3.18	3.20	3.41	3.75	3.95	4.20	4.30	4.65	4.72	5.00	5.00	274
1.83	2.21	2.09	2.51	2.16	2.68	2.20	2.78	2.28	2.93	2.30	3.00	2.32	3.10	2.32	3.20	275
1.96	2.00	2.23	2.23	2.33	2.35	2.41	2.41	2.55	2.55	2.61	2.63	2.71	2.71	2.75	2.75	276
1.95	2.41	2.60	3.05	3.05	3.46	3.40	3.79	4.20	4.46	4.75	5.00	5.50	5.55	6.00	6.00	277
1.68	1.77	1.90	1.98	2.00	2.07	2.04	2.14	2.12	2.28	2.18	2.35	2.21	2.42	2.22	2.50	278
1.64	1.84	1.89	2.17	2.00	2.30	2.09	2.41	2.24	2.62	2.31	2.75	2.41	2.90	2.48	3.00	279
1.79	2.65	1.96	3.10	2.04	3.33	2.10	3.51	2.20	3.86	2.27	4.10	2.31	4.33	2.35	4.50	280
1.50	1.70	1.81	2.00	2.00	2.17	2.15	2.30	2.42	2.53	2.60	2.70	2.83	2.88	3.00	3.00	281
1.54	1.97	1.73	2.24	1.85	2.40	1.91	2.51	2.06	2.75	2.18	2.90	2.27	3.08	2.32	3.19	282
1.55	2.05	1.79	2.39	1.90	2.57	2.00	2.70	2.16	3.00	2.28	3.18	2.39	3.38	2.48	3.50	283

period June 1–September 30 in each year of record.
year of record.
stations for computation of average annual precipitation.

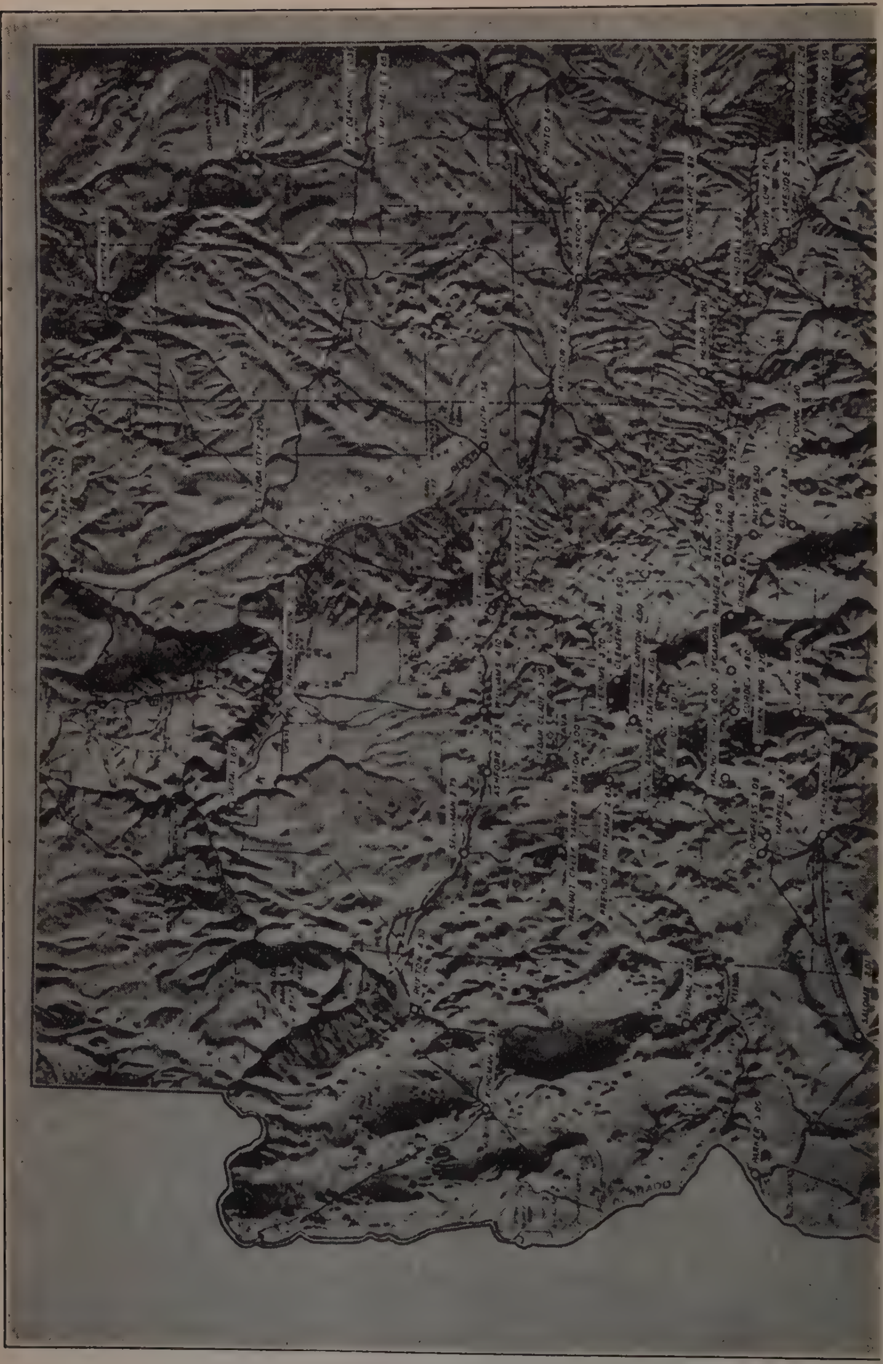
the highest peak discharge each year of record, the average of which he termed "average annual flood." In Table 3, the data on the average of highest rainfall each summer and of highest rainfall each year are comparable to Mr. Hazen's "average annual flood."

Isopluvial Maps

The isopluvial maps published by Mr. Yarnell[2] were based on five stations in the Southwestern States. In spite of the fact that the present analysis includes frequency curves for 283 stations in the same region, due to marked variation in frequency curves for closely adjacent stations of comparable records, isopluvial maps are difficult to construct. This difficulty can be attributed to a number of facts. Owing to the great divergence in topographic relief between closely adjacent points, two stations within a very short distance of one another may be in very different topographic settings. One may be in the middle of a broad, flat plain, the other in a deep, narrow mountain canyon. It appears from the present analysis that stations above El. 7,500 experience relatively few high rainfalls. This indicates that elevation above sea level has some effect on the frequency characteristics, although ruggedness of relief and position of the station in relation to the source of moisture and the surrounding mountainous areas probably are of more direct importance.

The small areal extent of high rainfalls occurring in summer-type storms is a factor leading to differences between stations since any high-intensity, individual storm rarely is recorded at more than one station.

The 24-hr rainfall quantity expected to be equaled or exceeded once in 50 years at various stations in relation to topography is shown by the relief maps


ST. JOHNS 2.22
HOLBROOK 2.58
WINSLOW
SNOWFLAKE
SHOW LOW
LAKESIDE
LEUPP 1.34
NATURAL BRIDGE 3.22
PAYSON 2.50
YOUNG
CHILDS
CLEMENCEAU
SYCAMORE RANGER STATION
ASHFORK 2.33
WILLIAMS 4.10
SELIGMAN
PRESCOTT DRY FARM
WALNUT CREEK RANGER STATION
CONGRESS
YARNELL
CORDES
YUMA
PARKER
SALOME
KINGMAN
COLORADO

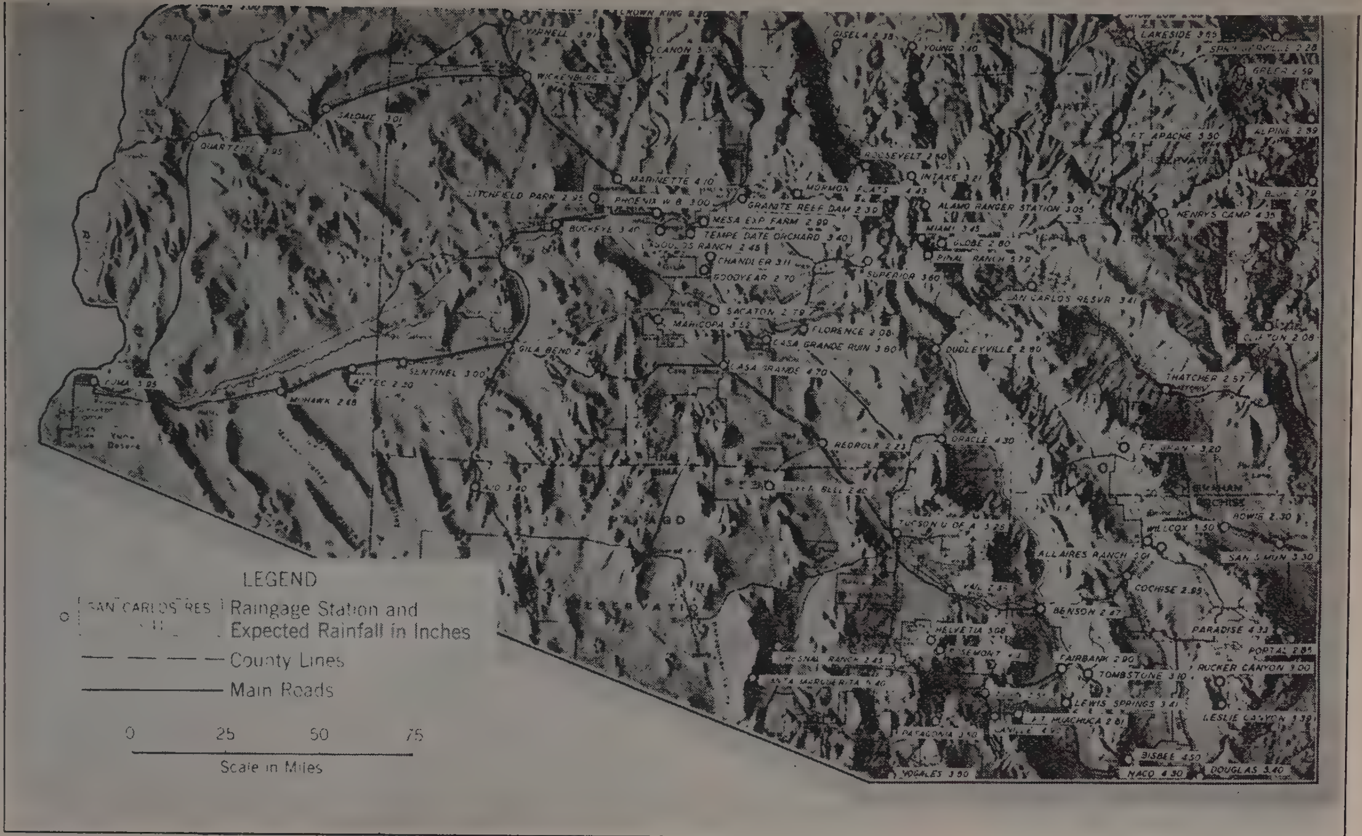
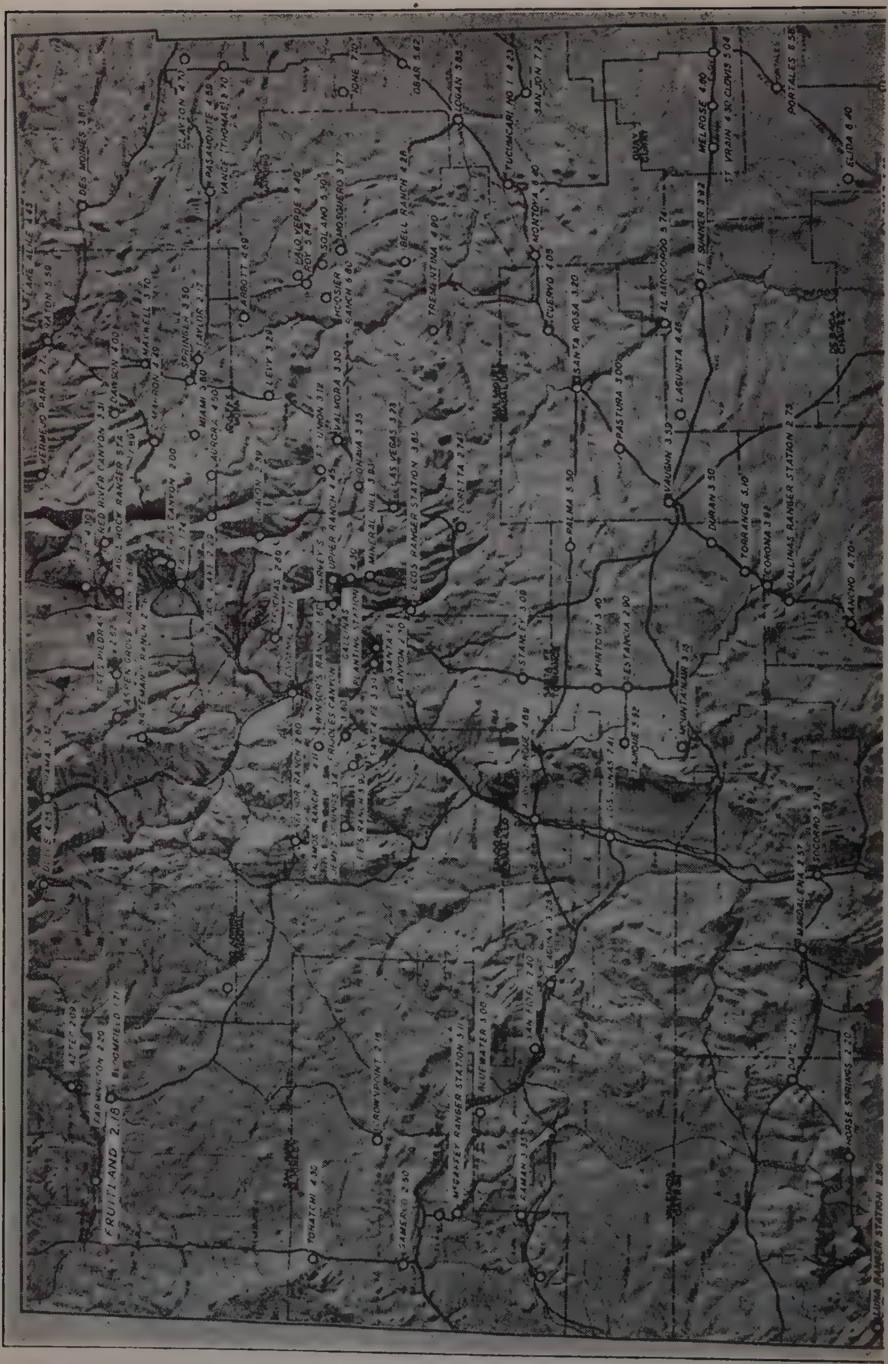


FIG. 3.—ARIZONA: 24-HR RAINFALL EXPECTED TO BE EQUALED OR EXCEEDED ONCE IN 50 YEARS


LAKE ALICE 4.43
RATON 5.59
DES MOINES 3.80
CLAYTON 4.73
PASAMONTE 4.89
VANCE (THOMAS) 5.70
IONE 7.70
OBAR 5.82
LOGAN 3.85
TUCUMCARI NO 1 4.23
SAN JON 7.73
MELROSE 4.90
ST VRAIN 4.30
CLOVIS 5.04
PORTALES 6.56
ELIDA 6.40
VERMEJO PARK
DAWSON 4.00
MAXWELL 3.70
CIMARRON 4.29
SPRINGER 3.50
TAYLOR
ABBOTT 4.60
PALO VERDE 4.40
ROY 5.84
SOLANO 5.30
MOSQUERO 3.77
HOOSIER RANCH 6.80
BELL RANCH 4.28
MONTOYA 4.90
TREMENTINA 4.90
CUERVO 4.05
ALAMOGORDO 5.74
FT. SUMNER 3.92
LEVY 3.29
MIAMI 3.80
AURORA 4.50
RED RIVER CANYON 3.51
EAGLE ROCK RANGER STA
SANTA ROSA 3.20
LAGUNITA 4.45
PASTURA 3.00
VAUGHN 3.39
DURAN 3.50
TORRANCE 5.10
CORONA 3.82
GALLINAS RANGER STATION 2.75
ANCHO 4.70
FT. UNION 3.12
VALMORA 3.30
UPPER RANCH 4.45
ROCIADA
LAS VEGAS 3.23
MINERAL HILL 3.83
PECOS RANGER STATION 3.85
GALLINAS PLANTING STATION
PALMA 3.30
STANLEY 3.00
McINTOSH 3.40
ESTANCIA 2.90
TAJIQUE 3.82
MOUNTAINAIR 3.15
SANTA FE 3.33
SANTA FE CANYON 2.90
WINSOR'S RANCH
FRIJOLES CANYON
ALAMOS RANCH
LOS LUNAS 1.71
LAGUNA 3.23
SAN FIDEL 2.80
MAGDALENA 2.37
SOCORRO 3.12
DATIL 2.11
HORSE SPRINGS 2.20
LUNA RANGER STATION 2.30
TRES PIEDRAS
ASPEN GROVE RANCH
BATEMAN RANCH
AZTEC 2.59
FARMINGTON 2.20
BLOOMFIELD 1.71
FRUITLAND 2.18
CROWNPOINT 2.18
BLUEWATER 3.00
McGAFFEY RANGER STATION 3.11
TOHATCHI 4.30
GAMERCO 2.50
RAMAH 3.35

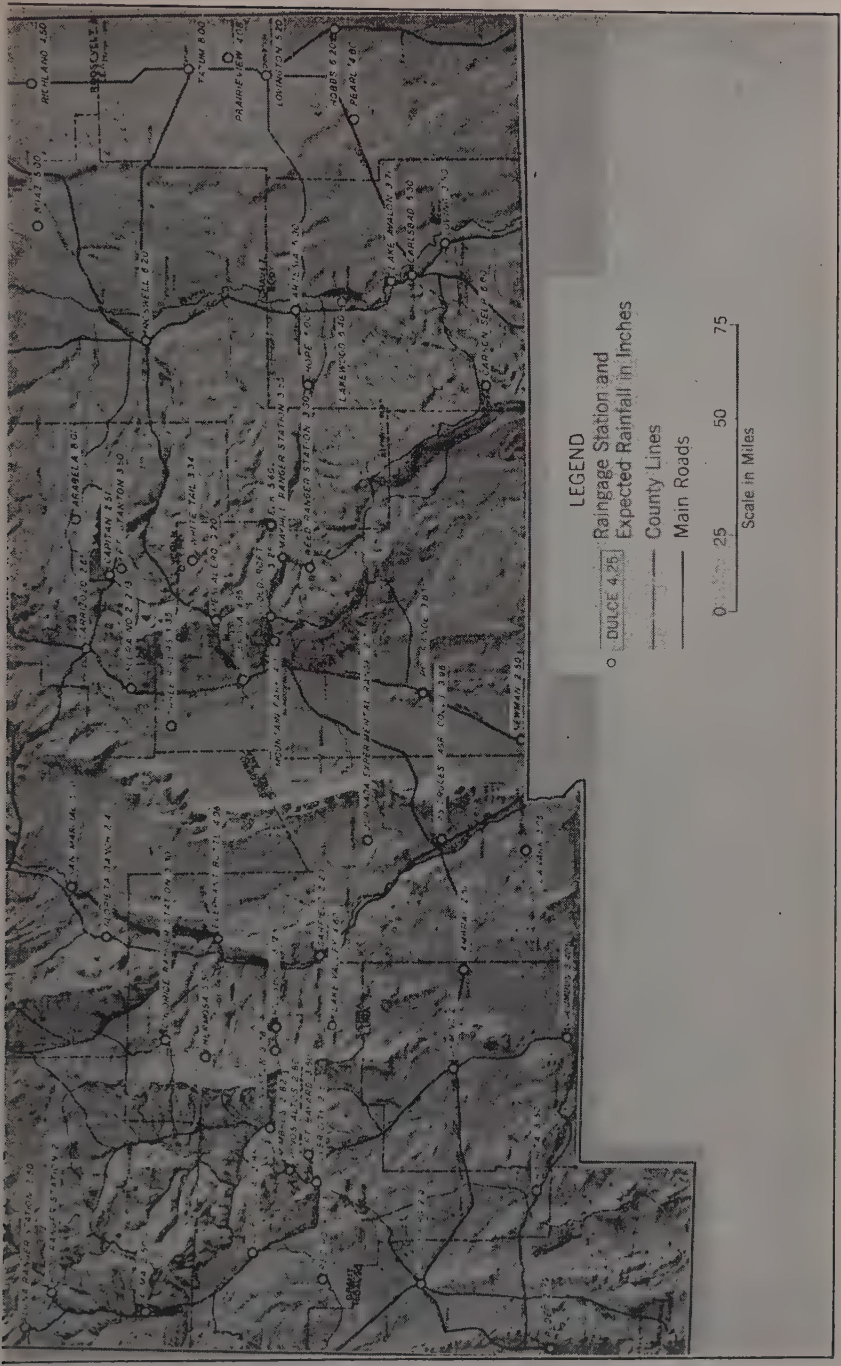


FIG. 4.—NEW MEXICO: 24-HR RAINFALL EXPECTED TO BE EQUALED OR EXCEEDED ONCE IN 50 YEARS

of Figs. 3 and 4. These maps demonstrate the variability of the recurrence-interval figure between closely adjacent stations, and they indicate the position of the stations relative to the major mountain masses. Stations in deep mountain valleys, for the most part, are shielded from the effect of moisture-bearing air masses moving into the area and tend to have low rainfall expectancy. This is not true in all cases, but the relief apparently offers sufficient explanation for the relative magnitude of the recurrence-interval figure for certain stations. In other cases, two stations in apparently similar topographic positions have quite different expectancies.

In spite of these differences between individual stations, certain general patterns of rainfall expectancies are apparent in New Mexico and Arizona. The area in southeastern New Mexico, particularly along the eastern flanks of the Delaware, Sacramento, and Capitan mountains, is subject to high rainfalls because summer moisture moving in from the south or southwest impinges on these topographic barriers. The same is true for northeastern New Mexico: In the central Pecos watershed no important topographic barrier obstructs the moisture-bearing winds south of the relatively rugged country in the headwaters of the Pecos and Canadian rivers.

The Black Range between Silver City and Hillsboro in southwestern New Mexico forms a barrier to south winds and protects northwestern New Mexico from high rainfalls. Stations in the Rio Grande Valley as far north as Albuquerque, N. Mex., however, are subject to high rains from moisture moving up the valley.

In Arizona, the mountainous area of the central and northwestern portion, particularly along the Mogollon and Yarnell escarpments, are subject to high rainfalls and tend to protect or shadow the northeastern plateau country.

These relationships can be easily visualized by inspection of Figs. 3 and 4. From these generalized relations an isopluvial map for 50-yr frequency has been drawn and is presented in Fig. 5. No attempt was made in the preparation of this map to draw each "isopluve" in the exact position indicated by the computed value for each individual station, but the lines were generalized to indicate broadly the comparative expectancies of various regions.

In the use of the recurrence-interval data presented herewith, engineers are cautioned to compare the topography and position of the point for which a recurrence-interval value is desired with those factors affecting all stations near the particular point. Moreover, it must be remembered that the data presented are for a 24-hr period, but in the summer type of rainfalls, the maximum rainfall in 24 hr probably fell in a much shorter period of time. The value of total storm rainfall for a given recurrence-interval in the summer period may be used in connection with average intensity-time patterns for the purpose of designing engineering structures.

APPLICABILITY OF STATION-YEAR ANALYSIS

One method of ironing out differences between recurrence-interval curves of adjacent stations and improving the value of short records is a combination of stations by the station-year method. The requirements for use of this method discussed by H. C. S. Thom[7] and Katharine Clarke-Hafstad[8] may be summarized as follows:

(1) Stations grouped must be mutually independent—that is, far enough apart so that no single storm covers more than one of the stations; and

(2) Stations grouped must be included in a meterologically homogeneous region.

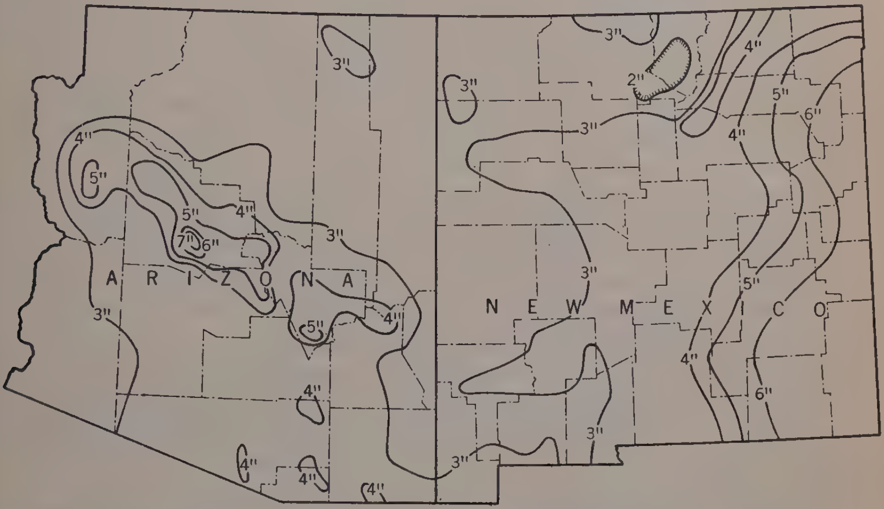


FIG. 5.—ISOPLUVIAL MAP OF ARIZONA AND NEW MEXICO (24-HR RAIN EQUALED OR EXCEEDED ONCE IN 50 YEARS)

Mr. Thom defines a meteorologically homogeneous region as an area each point of which would have identical frequency curves of rainfall. He states that delineation of such a region by this criterion is impractical since "assignable causes," such as topographic differences, operate to produce dissimilarity of rainfall experience at various points within the region; and he suggests, therefore, that homogeneity must be established by meteorological studies. Assignable causes operate in the following way: A storm situation, one favorable for rain, prevailing over a region may be so weak that rainfall occurs only at points that are subject to significant orographic effects, and these stations, over a long period, would therefore experience more rainy days than do stations with assignable causes of lesser magnitude.

Although it is possible, in New Mexico and Arizona, to delineate meteorologically homogeneous areas defined by meteorologic studies, recurrence-interval curves at various stations within such a region may be so different that assignable causes appear to operate in these states to such an extent that there may be more difference between curves of stations within the homogeneous area than between two stations in dissimilar meteorologic regions.

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[7] "On the Statistical Analysis of Rainfall Data," by Herbert C. S. Thom, *Transactions*, Am. Geophysical Union, Pt. II, 1940, pp. 490–499.

[8] "Reliability of the Station-Year Rainfall-Frequency Determinations," by Katharine Clarke-Hafstad, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 633.

Examples of station-year groupings were platted in connection with the present study. It did not appear satisfactory to group stations by the station-year method, although the stations were in meteorologically similar regions, inasmuch as the frequency curves were so distinctly different.

COMPARISON OF MAGNITUDE OF 1-DAY, 2-DAY, AND 3-DAY RAINS

In the use of frequency-magnitude data for design, it usually is necessary to apply an intensity pattern to a given rainfall total. In general, it may be stated that in New Mexico and Arizona summer rainfall depths published as 24-hr totals characteristically fall in a period much less than 24 hr—usually less than 10 hr. Since design storms ordinarily are assigned an intensity pattern that would result in relatively high stream discharge, the rainfall magnitudes for summer listed in Table 3 logically may be used with the characteristic intensity patterns presented in Fig. 1(*a*).

The great winter floods in these states are the result of rainfalls extending over a period of days, as was the case in the flood of the upper Gila River in January, 1916. For the determination of the probability of long-duration storms, the recurrence-interval of various total amounts of precipitation in periods greater than 24 hr was derived by tabulating the highest rainfall amounts each year that fell on two or three consecutive days, arranging the terms in order of magnitude, and platting as in the studies of 24-hr rains. These arrays were prepared for a limited number of stations, each of which was chosen to represent a certain area with characteristic topographic relief and elevation.

This analysis showed that the ratio of the total rainfall in a 48-hr period to that falling in a 24-hr period is essentially constant regardless of the recurrence-interval of the rain, and similarly, the ratio of 72-hr total to 24-hr is approximately constant. At any given recurrence-interval the amount of rain in any consecutive 48-hr period is about 10% to 20% greater than the 24-hr amount expected, and the total for any 72 consecutive-hr period is 20% to 30% greater than the amount in 24 hr.

These ratios are only an indication of the relation between 1-day, 2-day, and 3-day rains, since the present analysis did not include sufficiently detailed studies of long-duration rains and winter intensity patterns to form the basis for positive statements on this subject. The rains experienced in the Pecos River watershed of eastern New Mexico in September, 1941, indicate that, at least in that section of the state, summer-type rainfalls of very high amounts may fall on two consecutive days.

ACKNOWLEDGMENTS

The writer gratefully acknowledges the assistance of Thomas Maddock, Jr., for suggestions during the progress of the study, B. T. Mitchell in checking original records, and a number of workers of the Work Projects Administration, principally the late E. F. Blanchard, who copied data from the original observers' records and assisted in the frequency tabulations. The cooperation of E. L. Hardy and G. K. Greening, U. S. Weather Bureau, in allowing access to the original records of cooperative observers is appreciated.

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PAPERS

EFFECT OF TURBULENCE ON SEDIMENTATION

By William E. Dobbins,[1] Jun. Am. Soc. C. E.

Synopsis

The basic theory of turbulent flow has been applied successfully to explain the observed vertical distribution of suspended matter in streams in which the suspended load is in equilibrium with the bed material. This paper presents the results of studies made in an effort to extend the theory to explain the concentration changes in a stream or settling basin under nonequilibrium conditions. The general differential equation expressing the concentration changes during turbulent sedimentation in an infinitely wide stream is derived, and the complete solution for a certain simplified case is presented. Evidence of the validity of this solution is provided by the results of experiments which are described herein.

Introduction

An attempt to set up a mathematical theory of sedimentation should take into account both the character of the sediment and the character of the fluid motion.

The sediment characteristics to be considered are not only the settling velocities of the individual particles but also the question of whether or not they retain their individualities as they settle. When two particles of a nonflocculent material come together the only effect is the momentary mutual interference; if two particles of a flocculent material come together they may lose their individual identities and become one. Thus, in a suspension of nonflocculent material, the sediment characteristics are constant, whereas in a suspension of flocculent material they are continuously changing.

The characteristics of the fluid motion under consideration are not only the motion of the fluid as a whole but also the momentary fluctuations from the average motion at every point in the fluid mass. To take into account the motion of the fluid as a whole, the boundaries of the flow, the average point velocities and the variations of these with time should be studied. Besides

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Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **July 1, 1943.**

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these factors, all secondary motions, such as convection or displacement currents and the fluctuations which are characteristic of turbulent flow, must be considered if the fluid motion is to be completely defined.

A complication arises through the fact that there is not complete independence between the sediment and the fluid flow characteristics. In the case of a flocculent material, the rate at which and the extent to which coagulation takes place are influenced by the nature of the flow. Any mixing of the fluid will increase the chances of contact between the individual particles, and hence the speed of coagulation. On the other hand, the shear stresses set up in the particles by the velocity gradients tend to limit the size to which the particles can grow.

In a horizontal flow tank, any upward flow caused by a secondary motion at some point must be compensated by a downward flow at the same rate at some other point. This is obviously necessary if the fluid mass is to remain continuous. In a suspension of particles which are settling out, the suspended matter tends to become more concentrated toward the bottom than above, although the suspension may be clearing throughout the entire depth. An upward flow, then, carries fluid from a region in which the concentration of suspended matter is high into one in which it is relatively low. Likewise, a downward current carries fluid with a low concentration into a region where the concentration is high. Thus, one effect of the secondary motion is to create an upward transport which tends to retard the rate at which the suspension clears. On the other hand, the secondary motion might increase the rate of coagulation and there also tend to increase the rate of clearance. It is clear, then, that whether or not a suspension of flocculent material will settle more rapidly in a turbulent fluid than in a quiescent fluid will depend upon the relative magnitudes of the effects of the turbulence in increasing the rate of coagulation and in transporting the material upward. In a suspension of discrete particles, the upward transport is the only effect of turbulence and hence the rate of clearance is retarded.

The complexities involved in the analysis of the secondary motions themselves combined with those involved in the analysis of the coagulation phenomenon make the analysis of the sedimentation problem very difficult. However, an understanding of the separate effects of secondary motions and of coagulation on sedimentation would be helpful in understanding the entire phenomenon.

One of the earliest and most widely known attempts to set up a mathematical theory for settling in a sedimentation basin was made by the late Allen Hazen, M. Am. Soc. C. E. (1).[2] He distinguished between quiescent and "turbulent" flow and restricted his analysis to suspensions of discrete particles having a single settling velocity. The principal contribution which he made was to demonstrate that the extent of the removal of discrete particles in a quiescent basin is independent of the depth of the basin. His analysis of sedimentation in turbulent basins is incorrect because it is based on incorrect assumptions.

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[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix 1).

The case of sedimentation of discrete particles of varying settling velocities in a quiescent basin has been presented by T. R. Camp, M. Am. Soc. C. E. (2).

The study described herein concerns the case of the sedimentation of discrete particles of uniform size in a stream in which turbulence is fully developed. By "fully developed turbulence" is meant a condition such that, although the velocity at every point is in a continuous state of flux, there is a statistical average velocity that remains constant. Modern developments in fluid mechanics have led to an understanding of the phenomenon of turbulence which has enabled a rational attack to be made on the sedimentation problem.

BASIC THEORY OF SEDIMENT SUSPENSION BY TURBULENCE

Notation.—The letter symbols introduced in this paper are defined where they first appear and are assembled for convenience of reference in Appendix 2.

The Principle of Turbulent Transport.—In the modern theory of turbulence it has been shown that the intensity of shear τ at a point in a moving fluid is

$$\tau = \rho \left| \bar{v}'_y \right| l \frac{\partial \bar{v}_x}{\partial y} \qquad (1)$$

in which ρ is the fluid density; l is the Prandtl (3) "mixing length"; $\left| \bar{v}'_y \right|$ is the absolute magnitude without regard to sign of the temporal mean value of the fluctuation velocity in the y-direction; and $\frac{\partial \bar{v}_x}{\partial y}$ is the temporal mean velocity gradient. Eq. 1 expresses the fact that the rate of transport of momentum per unit volume across any stream line is proportional to the gradient of the momentum per unit volume in the direction of the transfer. Theodor von Kármán, M. Am. Soc. C. E. (4), showed that the rate of transport of any local characteristic of the fluid, such as its heat content, salinity, or silt content, can be expressed by an equation similar to Eq. 1. If c represents the magnitude per unit volume of such a characteristic and $\left| \bar{v}'_s \right|$ is the mean numerical value of the velocity fluctuations of the fluid in any direction s, the rate of transport of the local characteristic in the direction of s will be given by

$$-\beta \left| \bar{v}'_s \right| l \frac{\partial c}{\partial s} = -\epsilon \frac{\partial c}{\partial s} \qquad (2)$$

the minus sign indicating that the net transport is in the direction of decreasing c, and β being a factor whose value depends upon the characteristic being considered. The quantity ϵ is called the exchange or mixing coefficient. If c represents the momentum per unit volume and s is taken in the direction normal to the flow, the value of β is unity.

If c represents the magnitude or intensity of some inherent characteristic of the fluid, such as its heat content or color, it seems likely that the value of β would also be unity, since the characteristic would be transported by the transport of the fluid itself. This has been confirmed by A. A. Kalinske, Assoc. M. Am. Soc. C. E., and J. M. Robertson, Jun. Am. Soc. C. E. (5), in studying the diffusion of droplets of dye which had the same density as the water in which the diffusion took place.

In case c is the concentration of sediment in suspension, it is to be expected that the value of β might differ somewhat from unity, because the presence of the sediment increases the density of the fluid as a whole and thus alters the pattern of the turbulence, and also because the mixing of the suspended particles might not be the same as for the fluid masses. In studying suspensions of sand grains in water, Hunter Rouse, M. Am. Soc. C. E. (6), found that the value of β was the same for all suspensions in which the grain sizes were less than $\frac{1}{16}$ mm in diameter but that it changed for suspensions of larger grains. He was dealing with concentrations up to about 3% by weight. Since he found no systematic deviations in the value of β for material of any one size when the concentrations were varied, he attributed the variations with grain size to the fact that the mixing of the larger grains was not the same as that of the fluid itself. He had no means of determining the actual values of β, but simply the degrees of variation. Experiments by V. A. Vanoni, Assoc. M. Am. Soc. C. E. (7), indicated a value of β close to unity when dealing with relatively low concentrations of small grains. It is probable that the value of β can be taken as unity in most practical cases.

The Theory of Sediment Suspension.—In a turbulent stream the eddies dig into the bed and carry material into suspension. This action is opposed by the settling of the particles due to their own weight. If steady conditions prevail, there will be an equilibrium between the rate at which sediment is raised from the bed by the turbulence and the rate at which it settles on to the bed again. At every elevation above the bed, a similar equilibrium will result between the rate of upward transport due to the turbulence and the rate of fall due to the weight of the particles. Thus the concentration will remain constant at every elevation, although the particles of which the suspension is composed will be changing continuously.

If the concentration at any elevation, y, be denoted by c, and the settling velocity of the particles by w, the rate at which the material settles through a unit area at the elevation y is $w\,c$. If the state of equilibrium exists, this rate will be balanced by the rate of turbulent transport upward, and

$$w\,c = -\,\epsilon\,\frac{\partial c}{\partial y} \qquad \text{(3)}$$

This equation was first used by W. Schmidt (8) in studying the distribution of dust particles in air and was applied to the problem of silt suspension by Morrough P. O'Brien, M. Am. Soc. C. E. (9). Integration of Eq. 3 yields an expression for the concentration, c, in terms of the concentration, c_a, at some arbitrary elevation, a:

$$\log_e \frac{c}{c_a} = -\,w \int_a^y \frac{dy}{\epsilon} \qquad \text{(4)}$$

To evaluate the integral in Eq. 4 it is necessary to express ϵ as a function of y.

The general validity of Eq. 4 was established by Professor Rouse (6) in experiments in which a uniform intensity of turbulence was imposed on a suspension of sand in water by means of an agitating mechanism. Under such conditions, ϵ is independent of y and Eq. 4 becomes

$$c = c_a \, e^{-w\epsilon^{-1}(y-a)} \qquad (5a)$$

in which e is the base of Naperian logarithms. If it is assumed that ϵ is constant down to the mean level of the top of the bed material, a can be assumed zero and Eq. 4 reduces to

$$c = A^{-wy\epsilon^{-1}} \qquad (5b)$$

in which A is the concentration at $y = 0$. In such a distribution, the total sediment, W, held in suspension per unit area between any two elevations, y_1 and y_2, is given by the expression

$$W = \int_{y_1}^{y_2} c \, dy = \frac{c_1 \, \epsilon}{w} \left(1 - \frac{c_2}{c_1} \right) \qquad (6)$$

If the values of the concentrations at any two elevations, y_1 and y_2, are known, the value of $\frac{w}{\epsilon}$ can be computed from the equation,

$$\frac{w}{\epsilon} = \frac{1}{y_2 - y_1} \log_e \frac{c_2}{c_1} \qquad (7)$$

In an actual stream, ϵ varies with the distance from the bed and also from point to point across the section. If the flow is unsteady and nonuniform it will also vary from section to section and from time to time. If the discussion is limited to the case of steady uniform flow in the central region of a very wide stream, ϵ will be a function of y alone. In such a case, the value of the shear, τ, at any distance y from the bottom is given by the expression,

$$\tau = \tau_0 \left(1 - \frac{y}{h} \right) \qquad (8)$$

in which h is the total depth and τ_0 is the bottom shear. If this is combined with Eq. 1—

$$\epsilon = \beta \, \frac{\tau_0}{\rho} \, \frac{1 - \dfrac{y}{h}}{\dfrac{dV}{dy}} \qquad (9)$$

in which $\bar{v}_x$ has been replaced by V. It has been shown by G. H. Keulegan (10) and also by Mr. Vanoni (11) that the von Kármán universal logarithmic velocity law for pipes can be applied to the case of uniform two-dimensional open-channel flow, yielding the expression

$$V = \overline{V} + \frac{1}{\kappa} \sqrt{\frac{\tau_0}{\rho}} \left(1 + \log_e \frac{y}{h} \right) \qquad (10a)$$

from which

$$\frac{dV}{dy} = \frac{1}{\kappa} \sqrt{\frac{\tau_0}{\rho}} \, \frac{1}{y} \qquad (10b)$$

Using this value for the velocity gradient, Eq. 9 becomes

$$\epsilon = \beta\,\kappa\sqrt{\frac{\tau_0}{\rho}}\left(1-\frac{y}{h}\right)y = \beta\,\kappa\,\bar{V}\sqrt{\frac{f}{8}}\left(1-\frac{y}{h}\right)y$$

$$= \beta\,\kappa\sqrt{g\,h\,S}\left(1-\frac{y}{h}\right)y \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

In these expressions, κ is the von Kármán universal constant having a value approximately equal to 0.40; $\bar{V}$ is the mean velocity of flow; f is the Darcy friction factor for the bed; g is the acceleration of gravity; and S is the slope of the energy gradient.

Eq. 11 indicates a value of ϵ which is a maximum at $y = \frac{h}{2}$ and which approaches zero as y approaches both zero and h. This general trend has been verified by Messrs. Kalinske and Robertson (5), who made measurements of ϵ by observing the spread of droplets of dye which were injected into a rectangular channel at various points in the cross section. If the value of ϵ, as given by Eq. 11, is used in Eq. 4, the expression for the distribution of sediment becomes

$$\frac{c}{c_a} = \left(\frac{\frac{h}{y} - 1}{\frac{h}{a} - 1}\right)^z \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which

$$z = \frac{w}{\kappa\,\beta\sqrt{\frac{\tau_0}{\rho}}} = \frac{w}{\kappa\,\beta\,\bar{V}\sqrt{\frac{f}{8}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Experiments by Mr. Vanoni (7) have indicated close agreement between the actual distribution in a turbulent channel and the distribution as expressed by Eqs. 12 and 13 when the values of κ and β were taken as 0.40 and unity, respectively. He found that the agreement was close throughout the major range of the depth.

It would seem that the value of ϵ cannot be absolutely zero at either the bed or the surface because a value of zero at the surface would indicate that the surface is absolutely smooth, and a value of zero at the bed would indicate no pickup. The experimental measurements of Messrs. Kalinske and Vanoni indicate, however, that ϵ follows the functional trend of Eq. 11 as close to the bed and the surface as measurements are practically possible.

The foregoing expressions refer the value of c at any elevation, y, to its value at some arbitrary elevation, a. Thus, these expressions determine the shape of the concentration curves but give no indication of the absolute magnitudes. They also apply only to the case of sediments made up of particles having a single settling velocity.

The absolute magnitudes of the concentrations should depend upon the bottom conditions. In experiments described subsequently in this paper a uniform mixing was imposed upon a suspension of particles of a fairly uniform size in a cylindrical container. The resulting equilibrium concentration for a given depth and a given degree of mixing depended only upon the distance of the mixing apparatus from the bottom, provided that the bottom was completely covered with the sediment when the equilibrium condition was reached.

Although many variables determine the flow pattern in a stream, a given combination of these variables will result in a definite flow condition. Hence, it seems logical to assume that there will be a definite rate of pickup of a given type of sediment. At the time of this writing the analysis of the problem has not progressed to the point where the rate of pickup can be expressed as a function of the variables involved.

Development of the Theory of Sedimentation for Turbulent Conditions

Statement of the Problem.—The equilibrium condition described in the previous chapter is a condition which is approached but is never fully reached. The silt content of a natural stream at some section might be more or less than the ultimate transport capacity because of a sudden change in the cross section immediately upstream. If steady uniform flow prevails downstream, sediment will be deposited or picked up as the new equilibrium condition is approached. In a settling tank the content of suspended matter at the inlet end is much greater than the carrying capacity, and so the material settles out. It is with the changes in concentration from section to section that this study is mainly concerned. The discussion is confined to the two-dimensional case corresponding to the conditions of steady uniform flow in the central region of a wide channel.

Derivation of the Differential Equation for Sedimentation in a Turbulent Stream.—With reference to Fig. 1, the x-axis is taken at the bottom of the channel in the direction of the flow. For the sake of generality, the concentration is assumed to increase with both x and y and the velocity with y. Consider the point m (x,y) and the element mm′nn′ having the dimensions dx and dy in the x,y-plane and unity in the normal direction, and let c be the concentration of suspended matter at any time t at the point m.

If the amount of suspended matter carried out of the element across the face nn′ by turbulence and settling and across the face m′n′ by turbulence and the flowing stream is subtracted from the weight carried into the element across the opposite faces, it may be shown that

$$\frac{dW}{dx\,dy\,dt} = \epsilon_y \frac{\partial^2 c}{\partial y^2} + \frac{\partial \epsilon_y}{\partial y}\left(\frac{\partial c}{\partial y} + \frac{\partial^2 c}{\partial y^2}dy + \frac{\partial^2 c}{\partial x\,\partial y}\frac{dx}{2}\right)$$
$$+ \epsilon_x \frac{\partial^2 c}{\partial x^2} + \frac{\partial \epsilon_x}{\partial y}\frac{\partial^2 c}{\partial x^2}\frac{dy}{2} + w\frac{\partial c}{\partial y} - \left(V + \frac{\partial V}{\partial y}\frac{dy}{2}\right)\frac{\partial c}{\partial x} \;\ldots\ldots (14a)$$

in which dW is the net increased amount of suspended matter in the element in time dt.

For the steady state, the concentration at a fixed point remains unchanged with time. Therefore, $\frac{dW}{dx\,dy\,dt} = 0$. The other terms which contain the factors dy and dx also become zero as the element $dx\,dy$ approaches a point. Then the final equation becomes

$$V\frac{\partial c}{\partial x} = \epsilon_y \frac{\partial^2 c}{\partial y^2} + \left(w + \frac{\partial \epsilon_y}{\partial y}\right)\frac{\partial c}{\partial y} + \epsilon_x \frac{\partial^2 c}{\partial x^2} \qquad \text{(14b)}$$

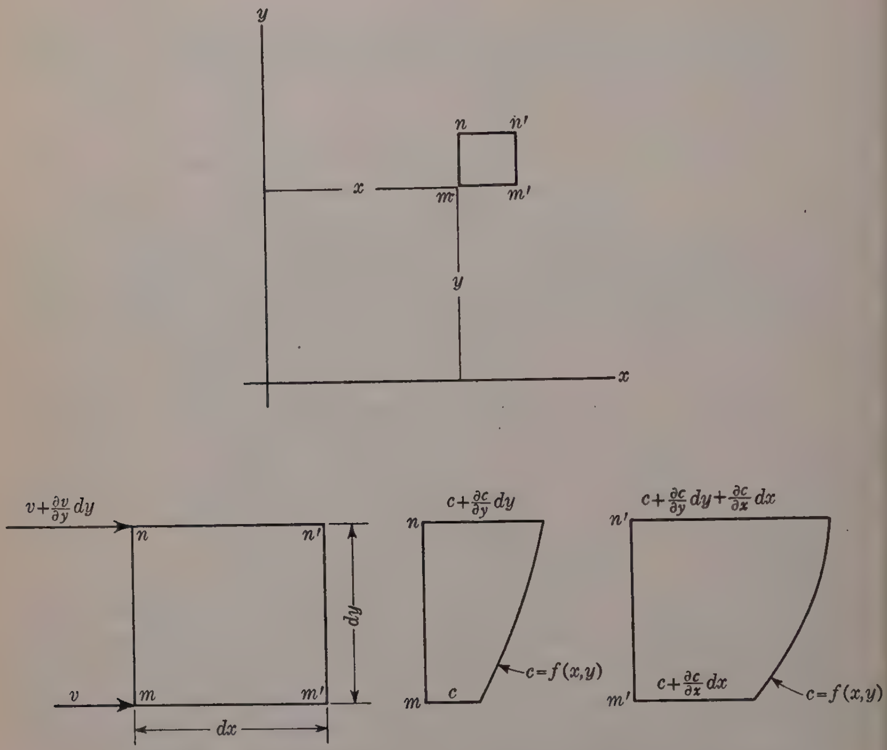
y
n
n'
x
m
m'
y
x
$v+\frac{\partial v}{\partial y}dy$
n
n'
dy
v
m
m'
dx
$c+\frac{\partial c}{\partial y}dy$
n
c=f(x,y)
m
c
$c+\frac{\partial c}{\partial y}dy+\frac{\partial c}{\partial x}dx$
n'
$c+\frac{\partial c}{\partial x}dx$
m'
c=f(x,y)

FIG. 1.—CONCENTRATION GRADIENTS IN THE CENTRAL REGION OF A WIDE CHANNEL

Eq. 14*b* is the general differential equation for the concentration changes in the two-dimensional case. An equation similar to 14*b*, but which does not take into account the term $\frac{\partial \epsilon_y}{\partial y}\frac{\partial c}{\partial y}$, has been presented by Professor Kalinske (14). Before a solution for Eq. 14*b* can be found, the factors V, ϵ_y, and ϵ_x must be expressed as functions of y. Eq. 10*a* expresses the necessary relationship for V and Eq. 11 gives the proper function for ϵ_y. It has already been stated that the value of ϵ_x is probably equal to or at least proportional to ϵ_y.

Simplifications of the General Differential Equation.—No solution has been obtained for the general two-dimensional case as expressed in Eq. 14*b*. However, the equation can be greatly simplified by making certain assumptions. If the velocity distribution were assumed to follow a parabolic law instead of the logarithmic law expressed in Eq. 10*a*, the velocity equation could be written (see Fig. 2).

$$V = V_s - k\,(h - y)^2 \quad \text{.................. (15a)}$$

in which V_s is the velocity at the surface and k is the constant of the parabola. Then

$$\frac{dV}{dy} = 2\,k\,h\left(1 - \frac{y}{h}\right) \quad \text{.. (15b)}$$

From Eq. 9, assuming β to be unity,

$$\epsilon_y = \frac{\tau_0}{\rho}\,\frac{1 - \dfrac{y}{h}}{\dfrac{dV}{dy}} \quad \text{..... (16a)}$$

Combining Eqs. 15*b* and 16*a* gives

$$\epsilon_y = \frac{\tau_0}{2\,k\,h\,\rho} \quad \text{...... (16b)}$$

which is a constant for any particular flow condition. If it is further assumed that $\epsilon_x \dfrac{\partial^2 c}{\partial x^2} = 0$ (that is, there is no diffusion of the suspended matter in the x-direction), the differential equation reduces to

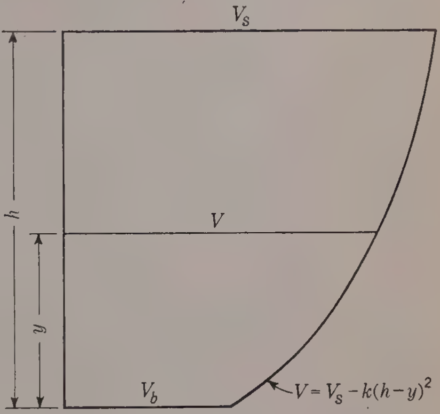

FIG. 2.—PARABOLIC VELOCITY DISTRIBUTION

$$[V_s - k\,(h - y)^2]\,\frac{\partial c}{\partial x} = \epsilon_y\,\frac{\partial^2 c}{\partial y^2} + w\,\frac{\partial c}{\partial y} \quad \text{............ (17a)}$$

A further simplification is obtained if it is assumed that V is constant throughout the depth, $\epsilon_x \dfrac{\partial^2 c}{\partial x^2}$ is zero, and ϵ_y is constant. In this case, Eq. 17*a* becomes

$$V\,\frac{\partial c}{\partial x} = \epsilon\,\frac{\partial^2 c}{\partial y^2} + w\,\frac{\partial c}{\partial y} \quad \text{.................... (17b)}$$

For the case in which the body of the liquid has no motion as a whole but has a uniform turbulence imposed on it throughout, the independent variables become y and t, and the differential equation becomes

$$\frac{dW}{dx\,dy\,dt} = \frac{\partial c}{\partial t} = \epsilon\,\frac{\partial^2 c}{\partial y^2} + w\,\frac{\partial c}{\partial y} \quad \text{................. (17c)}$$

Eqs. 17*b* and 17*c* are identical mathematically, because if V is constant, $V\,\dfrac{\partial c}{\partial x}$ is equivalent to $\dfrac{\partial c}{\partial t}$, in which t is equal to $\dfrac{x}{V}$.

It is to be realized that the simplifying assumptions involved in Eqs. 17 may introduce large errors when applied to the case of a natural stream. The assumption of a parabolic velocity distribution need not introduce large errors in the values of V but it does introduce considerable error in the value ϵ_y. The equation for this case was presented to show that the assumption of a constant value for ϵ_y is equivalent to assuming a parabolic velocity distribution. The assumption that the value of $\epsilon_x \dfrac{\partial^2 c}{\partial x^2}$ equals zero would not introduce a very large error if the concentration gradient in the x-direction were small compared to the gradient in the vertical direction. This would be the case when the concentrations do not differ greatly from the final equilibrium concentrations.

Boundary Conditions.—The solution for any practical case must satisfy not only the differential equation for that case but also the boundary conditions. The latter are four in number, two of which specify the conditions that prevail at the surface and at the bed, and two of which specify the original and the final distributions of the suspended matter. The rate of transport of sediment across any plane, $y = y_1$, is given by the expression

$$\text{Transportation rate} = \epsilon_y \left(\frac{\partial c}{\partial y} \right)_{y_1} + w\, c_{y_1} \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

There is no transport of material across the surface, in any case, and so the first boundary condition which must be satisfied is

$$\text{(I)} \ldots\ldots\ldots\ldots\ldots\ldots \left(\frac{\partial c}{\partial y} \right)_h = - \frac{w}{\epsilon_y} c_h$$

Since c_h varies with x or t, the concentration gradient at the surface is not a constant.

The rate at which material is picked up from the bottom by the turbulence is $- \epsilon_y \left(\dfrac{\partial c}{\partial y} \right)_{y=0}$. The minus sign accounts for the fact that the pickup is positive when the concentration gradient is negative. The rate of pickup of a given type of sediment depends only upon the characteristics of the flow and so, for steady uniform flow, the pickup will be constant. At the equilibrium condition the rate of pickup equals the rate of deposit, $w\, c_0$, and so the second boundary condition is

$$\text{(II)} \ldots\ldots\ldots\ldots \epsilon_y \left(\frac{\partial c}{\partial y} \right)_{y=0} = - w\, c_{\substack{y=0 \\ t=\infty}} = - w A$$

The third boundary condition is simply that the concentration at time $t = 0$ is some function of y between the limits of zero and h:

$$\text{(III)} \ldots\ldots c = f(y) \qquad \text{when} \qquad 0 < y < h, \qquad \text{and} \qquad t = 0$$

The fourth boundary condition is

(IV)............The concentration distribution approaches the equilibrium distribution as t approaches infinity.

For a channel this is expressed by Eq. 12 and for the case of a uniform turbulence throughout the fluid by Eqs. 5.

Solution of the Differential Equation for the One-Dimensional Case.—Eq. 17*c*, and its equivalent Eq. 17*b*, are the only forms of the differential equation which have yielded to an analytical solution. The problem in this case is to find a solution to the differential equation, Eq. 17*c*, in which ϵ and w are constants, that satisfies the boundary conditions as defined by I, II, III, and IV.

A particular solution to the differential equation can be obtained by assuming that the solution is a product of two functions—one a function of y alone and the other a function of t alone. Then

$$c = Y(y) \times T(t) \qquad (19)$$

By Eq. 17*c*,

$$\frac{T'}{T} = \frac{\epsilon\, Y'' + w\, Y'}{Y} \qquad (20)$$

Since each side of this equation is a function of only one of the independent variables, it can hold only if each member is equal to some constant, say $-k^2$. Then

$$T' + k^2\, T = 0 \qquad (21a)$$

and

$$Y'' + w\, Y' + k^2\, Y = 0 \qquad (21b)$$

The solution for T is found directly as

$$T = C_1\, e^{-k^2 t} \qquad (22)$$

in which C_1 is an arbitrary constant. The equation involving Y is a second-order linear differential equation which has the well-known solution

$$Y = e^{-0.5\, wy\epsilon^{-1}}\,(C_2 \cos \alpha\, y + C_3 \sin \alpha y) \qquad (23)$$

in which C_2 and C_3 are arbitrary constants, the exponent of e is $-\dfrac{w\,y}{2\,\epsilon}$, and

$$\alpha^2 = \frac{k^2}{\epsilon} - \frac{w^2}{4\,\epsilon^2} \qquad (24)$$

Then the equation,

$$c = e^{-k^2 t}\, e^{-0.5\, wy\epsilon^{-1}}\,(C_1 \cos \alpha\, y + C_2 \sin \alpha y) \qquad (25)$$

in which the constants are redefined, satisfies the differential equation.

It can be shown easily that the boundary condition for $t = \infty$ also satisfies Eq. 17*c*, and so

$$c = A\, e^{-wy\epsilon^{-1}} + e^{-k^2 t}\, e^{-0.5\, wy\epsilon^{-1}}\,(C_1 \cos \alpha\, y + C_2 \sin \alpha\, y) \qquad (26)$$

satisfies both the differential equation and the $t = \infty$ condition. In Eq. 26, A = the concentration at $y = 0$ and $t = \infty$.

To satisfy the boundary conditions for $y = 0$ and $y = h$ it is necessary to differentiate Eq. 26 with respect to y and to compare the result with conditions I and II. Performing the differentiation, there results:

$$\frac{\partial c}{\partial y} = -\frac{w}{\epsilon} A\, e^{-wy\epsilon^{-1}} + e^{-k^2 t}\, e^{-0.5\, wy\epsilon^{-1}} \times \left[\sin \alpha\, y \left(-C_1\, \alpha - \frac{w}{2\,\epsilon} C_2 \right) + \cos \alpha\, y \left(C_2\, \alpha - \frac{w}{2\,\epsilon} C_1 \right) \right] \qquad (27a)$$

When $y = 0$, this reduces to

$$\frac{\partial c}{\partial y} = -\frac{w}{\epsilon} A + e^{-k^2 t}\left(C_2\,\alpha - \frac{w}{2\,\epsilon} C_1\right) \qquad \text{(27}b\text{)}$$

By comparing this with the boundary condition for $y = 0$ (Condition II) it is seen that

$$C_2\,\alpha - \frac{w}{2\,\epsilon} C_1 = 0 \qquad \text{(28}a\text{)}$$

and

$$C_2 = \frac{w}{2\,\epsilon\,\alpha} C_1 \qquad \text{(28}b\text{)}$$

Comparing Eq. 27*a* with Condition I in which the value of C from Eq. 26 has been inserted, it is seen that

$$-\frac{w}{\epsilon}(C_1 \cos \alpha\, h + C_2 \sin \alpha\, h) = \left(C_2\,\alpha - \frac{w}{2\,\epsilon} C_1\right) \cos \alpha\, h - \left(C_1\,\alpha + \frac{w}{2\,\epsilon} C_2\right) \sin \alpha\, h \qquad \text{(29)}$$

from which

$$\tan \alpha\, h = \frac{C_2\,\alpha + \dfrac{w\,C_1}{2\,\epsilon}}{C_1\,\alpha - \dfrac{w\,C_2}{2\,\epsilon}} \qquad \text{(30)}$$

By combining Eq. 30 with Eq. 28 and changing it into a more convenient form, the following is obtained:

$$\eta = 2 \cot h\,\alpha = \frac{h\,\alpha}{\dfrac{w\,h}{2\,\epsilon}} - \frac{\dfrac{w\,h}{2\,\epsilon}}{h\,\alpha} \qquad \text{(31)}$$

Then, a solution of the differential equation which satisfies all of the boundary conditions except Condition III ($t = 0$) is given by

$$c = A\, e^{-wy\epsilon^{-1}} + e^{-0.5\,wy\epsilon^{-1}}\, e^{-k^2 t}\, C_1\left(\cos \alpha\, y + \frac{w}{2\,\epsilon\,\alpha} \sin \alpha\, y\right) \qquad \text{(32}a\text{)}$$

in which α is defined by Eq. 31, and k^2 is defined by Eq. 24. Inspection of Eq. 31 will reveal that for any given values of w, h, and ϵ there is an infinite number of real positive roots for α, any one of which could be used in Eq. 32*a*. Then, because of the linear nature of the differential equation, the most general solution is

$$c = A\, e^{-wy\epsilon^{-1}} + e^{-0.5\,wy\epsilon^{-1}} \times \sum_{n=1}^{\infty} e^{-(\alpha^2_n \epsilon + 0.25\,w^2 \epsilon^{-1})\,t}\, C_n\left(\cos \alpha_n\, y + \frac{w}{2\,\epsilon\,\alpha_n} \sin \alpha_n\, y\right) \qquad \text{(32}b\text{)}$$

To satisfy Condition III, C_n must now be chosen so that $c = f(y)$ when $t = 0$ and $0 < y < h$. Taking t as zero and c as $f(y)$ in Eq. 32*b*:

$$f(y) = A\, e^{-wy\epsilon^{-1}} + e^{-0.5\, wy\epsilon^{-1}} \sum_{n=1}^{\infty} C_n \left(\cos \alpha_n y + \frac{w}{2\, \epsilon\, \alpha_n} \sin \alpha_n y \right) \ldots (33a)$$

and

$$[f(y) - A\, e^{-wy\epsilon^{-1}}]\, e^{+0.5\, wy\epsilon^{-1}} = \sum_{n=1}^{\infty} C_n\, Y_n \ldots\ldots\ldots\ldots (33b)$$

in which

$$Y_n = \cos \alpha_n y + \frac{w}{2\, \epsilon\, \alpha_n} \sin \alpha_n y \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

The expression for C_n can be derived in a manner presented by H. S. Carslaw (15). The complete derivation for C_n which is on file in the Engineering Societies Library (see "Acknowledgments") yields the following:

$$C_n = \frac{2\, \alpha^2_n}{\left(\alpha^2_n + \frac{w^2}{4\, \epsilon^2} \right) h + \frac{w}{\epsilon}} \int_0^h [f(y) - A\, e^{-wy\epsilon^{-1}}]\, e^{0.5\, wy\epsilon^{-1}}\, Y_n\, dy \ldots (35)$$

Then the final solution to the problem is

$$c = A\, e^{-wy\epsilon^{-1}} + e^{-0.5\, wy\epsilon^{-1}} \sum_{n=1}^{\infty} e^{-(\alpha^2_n \epsilon + 0.25\, w^2 \epsilon^{-1})\, t}\, C_n\, Y_n \ldots\ldots (36)$$

in which Y_n is defined by Eq. 34, C_n by Eq. 35, and $\alpha_1, \alpha_2, \cdots \alpha_n$ are the successive real positive roots of the transcendental formula (Eq. 31).

The roots of Eq. 31 are found most easily by the graphical method illustrated in Fig. 3. The values of $h\, \alpha$ are plotted as abscissas and values of the expressions on each side of the equation as ordinates. The values of $h\, \alpha$ corresponding to the points of intersection of the curves are the values to be used in Eq. 36. It is to be noted that the curves of $2 \cot h\, \alpha$ corresponding to the successive values of α are the same for all cases, but that the proper intersecting curve to use depends upon the value of the ratio $\frac{w\, h}{2\, \epsilon}$. Since ϵ is a product of a velocity and a length, the ratio $\frac{w\, h}{2\, \epsilon}$ is dimensionless and Fig. 3 is a dimensionless plot.

Solutions of Eq. 36 for Various Cases.—If the distribution of sediment at time $t = 0$ is represented by the equation

$$f(y) = c_0\, e^{-By} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

in which c_0 is the concentration at $y = 0$, $t = 0$, and B is in general not equal to $\frac{w}{\epsilon}$, the expression for C_n becomes

$$C_n = \frac{2\, \alpha^2_n}{\left(\alpha^2_n + \frac{w^2}{4\, \epsilon^2} \right) h + \frac{w}{\epsilon}} \int_0^h (c_0\, e^{-By} - A\, e^{-wy\epsilon^{-1}})\, e^{0.5\, wy\epsilon^{-1}}\, Y_n\, dy \ldots (38)$$

The integral in Eq. 38 may be evaluated so that

$$C_n = \frac{2\,\alpha^2_n}{\left(\alpha^2_n + \dfrac{w^2}{4\,\epsilon^2}\right) h + \dfrac{w}{\epsilon}}$$

$$\times \left\{ \frac{c_0 \left[B + \left(\dfrac{w}{\epsilon} - B\right) e^{(0.5\, w \epsilon^{-1} - B) h}\, H_n \right]}{\left(\dfrac{w}{2\,\epsilon} - B\right)^2 + \alpha^2_n} - \frac{A\,\dfrac{w}{\epsilon}}{\alpha^2_n + \dfrac{w^2}{4\,\epsilon^2}} \right\} \ldots . (39a)$$

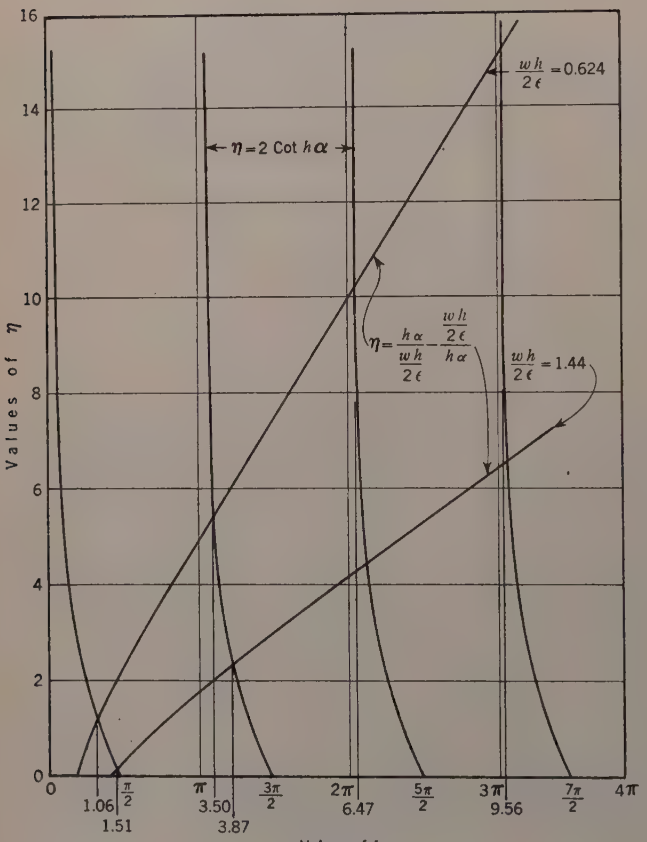

FIG. 3.—GRAPHICAL SOLUTION OF EQ. 31

in which

$$H_n = \cos \alpha_n h + \frac{w}{2\,\epsilon\,\alpha_n} \sin \alpha_n h \ldots\ldots\ldots\ldots\ldots (39b)$$

With the aid of Eq. 31, it can be shown readily that the value of H_n is ± 1, the plus sign applying for odd values of n and the minus for even values of n.

If the value of B is zero, the concentration at time $t = 0$ is uniform and equal to c_0. The expression for C_n in this case is

$$C_n = \frac{2\,\alpha^2_n\, w\,(c_0\, e^{0.5\, wh\epsilon^{-1}}\, H_n - A)}{\left(\alpha^2_n\,\epsilon + \dfrac{w^2}{4\,\epsilon}\right)\left[\left(\alpha^2_n + \dfrac{w^2}{4\,\epsilon^2}\right)h + \dfrac{w}{\epsilon}\right]} \quad \text{..........(40a)}$$

If $B = \dfrac{w}{\epsilon}$, Eq. 39a reduces to

$$C_n = \frac{2\,\alpha^2_n\, w\,(c_0 - A)}{\left(\alpha^2_n\,\epsilon + \dfrac{w}{4\,\epsilon}\right)\left[\left(\alpha^2_n + \dfrac{w^2}{4\,\epsilon^2}\right)h + \dfrac{w}{\epsilon}\right]} \quad \text{..........(40b)}$$

and Eq. 36 becomes

$$c = A\, e^{-wy\epsilon^{-1}} + (c_0 - A)\, e^{-0.5\, wy\epsilon^{-1}}$$

$$\times \sum_{n=1}^{\infty} \frac{2\,\alpha^2_n\, w\, e^{-(\alpha^2_n\epsilon + 0.25\, w^2\epsilon^{-1})\,t}\, Y_n}{\left(\alpha^2_n\,\epsilon + \dfrac{w^2}{4\,\epsilon}\right)\left[\left(\alpha^2_n + \dfrac{w^2}{4\,\epsilon^2}\right)h + \dfrac{w}{\epsilon}\right]} \quad \text{..........(41)}$$

Eq. 41, with c_0 taken as zero, is the solution for the case of scour in which the original concentration is zero for all values of y. With A taken as zero, it is the solution for the case of settling out from the equilibrium concentrations to zero concentration at all depths. It is to be noted that the solution represented by the complete equation is the sum of these two solutions.

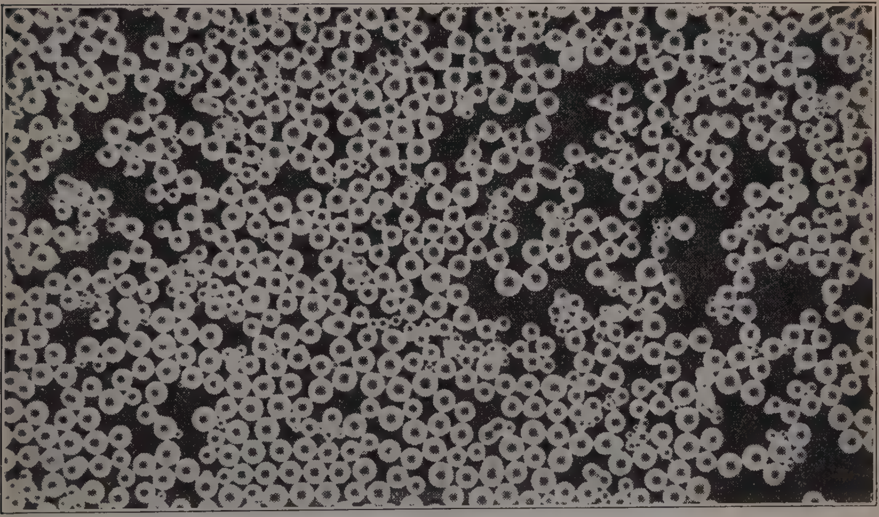

FIG. 4.—ENLARGED VIEW OF MOLDING POWDER PARTICLES

EXPERIMENTAL WORK

Scope and Purpose.—The experimental work was confined to a study of the one-dimensional case and had the primary object of verifying the equations developed in the previous section.

Description of the Suspensions Used in the Experiments.—The material used for the sediment in all of the experiments was the molding powder of a methyl methacrylate resin called "lucite." This material was obtained in the unplasticized and unlubricated state. The powder is made up of transparent spherical particles, most of them ranging in diameter from one tenth of a millimeter to one quarter of a millimeter. An average value of 1.194 g per cu cm was obtained for the density when measured by the pycnometer bottle method and by computation from the results of settling velocity measurements.

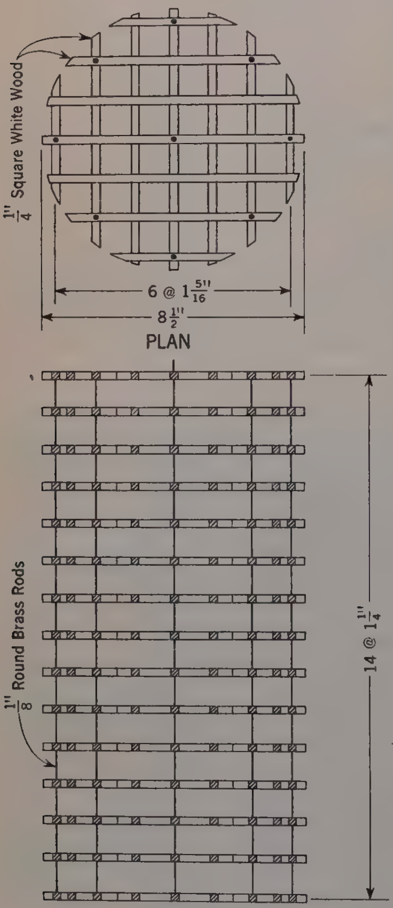

Fig. 5

Fig. 4 illustrates the material used in the experiments. This material, which was separated from the powder by sieves and a wind tunnel, was analyzed to determine the size and settling velocity distributions. The diameters of two hundred particles, which were chosen at random, were measured and the settling velocities in water at 10° C were then computed. In the computations the value of the drag coefficient was obtained from the equation by C. W. Oseen (16). The diameters varied from 0.138 mm to 0.216 mm and the corresponding settling velocities from 0.150 cm per sec to 0.344 cm per sec. The arithmetical average value of the settling velocity was 0.264 cm per sec. This value was used for w in all of the subsequent computations.

Suspensions of these particles in distilled water were used in all of the experiments. To prevent the particles from attaching themselves to air bubbles it was necessary to use a wetting agent. About 1 cu cm of an agent known as "Ultrawet No. 40" was used in each liter of the suspension. It was found that this amount had no measurable effect on the settling velocities of the particles.

Apparatus and Methods of Measurement.—The experimental setup was similar in its essentials to that used by Professor Rouse (6). In all of the runs the suspension was contained in a cylinder 9 in. in diameter by about 20 in. high. The mixing was accomplished by the lattice structure shown in Fig. 5. This was oscillated vertically in simple harmonic motion, thus producing an essentially uniform degree of mixing throughout the fluid. The cylinder was set in a thermostatically controlled water bath by which the temperature of the water in the cylinder was maintained at 10° ± 0.2° C.

The concentrations were measured by syphoning off samples of the suspension and obtaining the dry weight of the particles. In the sampling operation an amount equal to two or three times the volume of the siphon was wasted before the sample to be measured was collected. Each sample was collected in a 15-cc graduated centrifuge tube. When the lucite particles settled to the bottom of the tube they were picked up by an eye dropper and deposited in a small weighing dish which was then dried and weighed on an analytical balance.

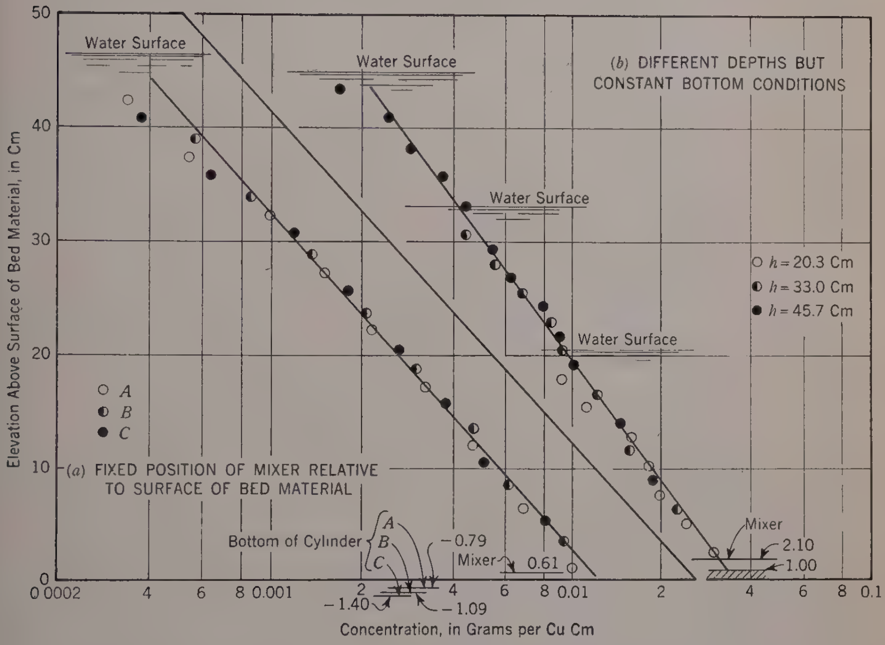

FIG. 6.—EQUILIBRIUM CONCENTRATIONS AT A MIXER SPEED OF 1.65 RPS

Procedure.—Measurements were made of the equilibrium concentrations with various total amounts of the powder, various mixing speeds, and various positions of the mixer with respect to the bottom of the cylinder. All of the results obtained were in agreement with the equilibrium distribution theory as expressed in Eq. 5*a*. Ample evidence of the validity of Eq. 5*a* has already been presented by Professor Rouse (6) and so all of the results will not be presented herein. Typical results are shown in Figs. 6 and 8. In each case the computed average thickness of the bottom deposit has been indicated after the equilibrium condition had been reached. The quantity of material on the bottom was determined as the difference between the volume in suspension and the total known amount used in the test. The volume in suspension was computed by Eqs. 6 and 7, the values of the necessary factors being taken directly from the linear plot. In these computations, the ratio of the dry weight to the volume occupied under water was taken as 1 g per 1.40 cu cm. This factor had been determined by measuring the volumes occupied by various known weights of material when settled out under water in a graduate.

Two runs were made for the purpose of checking the theory developed in the previous section. The first one was confined to the case in which the distribution of suspended matter at time $t = 0$ is defined by Eq. 5*b* and in which the pickup is zero. The solution for this case is given by Eq. 41 when A is taken as zero.

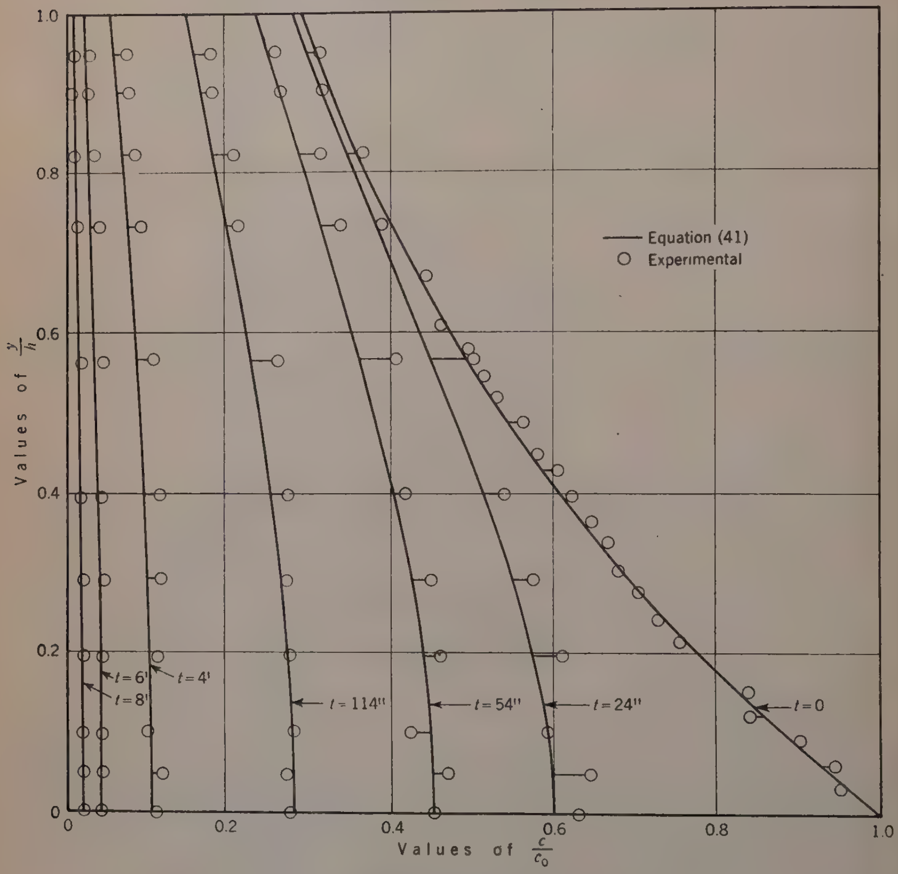

FIG. 7.—RUN IV. CONCENTRATION CHANGES WITH NO PICKUP; $\left(\frac{w}{\epsilon} = 0.030 \text{ cm}^{-1}; \text{ and } h = 41.6 \text{ cm}\right)$

To satisfy these conditions physically, it was necessary to impose the condition of no pickup suddenly upon the equilibrium condition. This was accomplished by the setup shown diagrammatically in Fig. 9. The cylinder was supported at the top, as shown in Fig. 9(*a*), with a clearance of about 2 in. between the bottom of the cylinder and the floor of the water bath. The grid shown in Fig. 9(*c*) rested on the bottom of the cylinder and was attached to a framework external to the cylinder. Sufficient lucite was added to fill the grid completely and extend about 1 cm above the top of the grid. When the mixer was set in motion, the lucite above the grid was picked up and distributed according to the logarithmic law. This satisfied the condition for the original distribution.

To impose the condition of no pickup suddenly, the supporting bars at the top were swung outward, allowing the cylinder to drop to the floor of the water bath. This was accomplished in a fraction of a second. The particles of lucite settled through the grid into the space below, from which they could not be picked up. At various times after the cylinder was dropped, samples were drawn off through the siphon. After most of the material had settled, the cylinder was lifted back to its original position, the siphon was set to a different value of y, and the entire process was repeated.

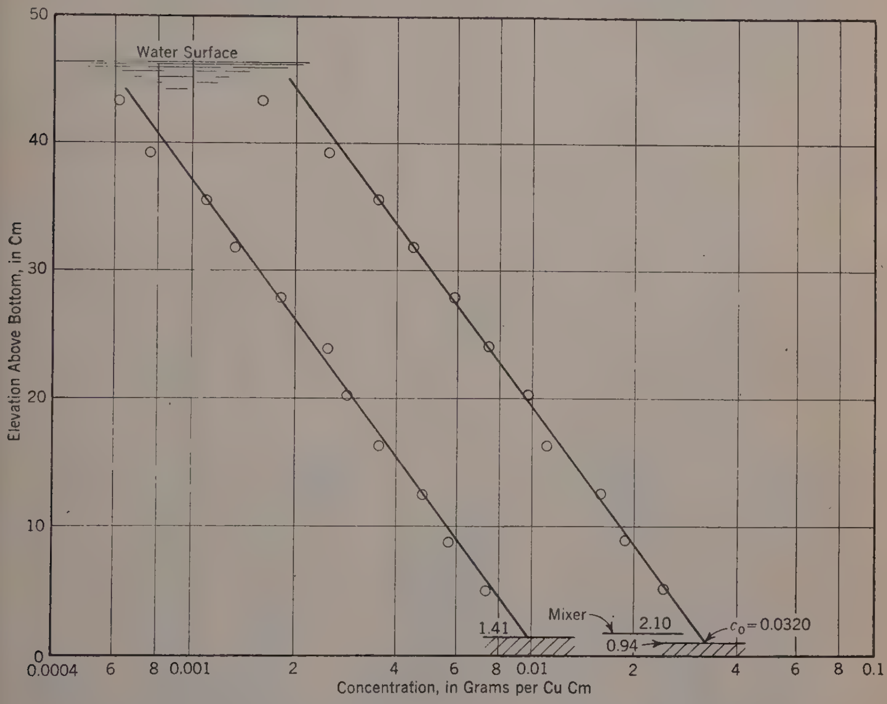

FIG. 8.—INITIAL AND FINAL EQUILIBRIUM CONCENTRATIONS; RUN V; MIXER SPEED, 1.65 RPS

The average sample size was about 14 cu cm and required about 6 sec for its collection. The volume of the siphon was found to be 13.8 cu cm and hence the time of passage through the siphon was also about 6 sec. To take this into account, the value recorded for the sampling time was 6 sec less than the time at which the center of the sample entered the collecting tube. A comparison between the theoretical and measured results is shown in Fig. 7.

The next run was made in order to check the theory for the case in which the pickup is not zero but has a constant value. An equilibrium condition was taken for the initial distribution of suspended matter, and another equilibrium condition with the same value of ϵ but with smaller absolute concentrations was taken as the final distribution.

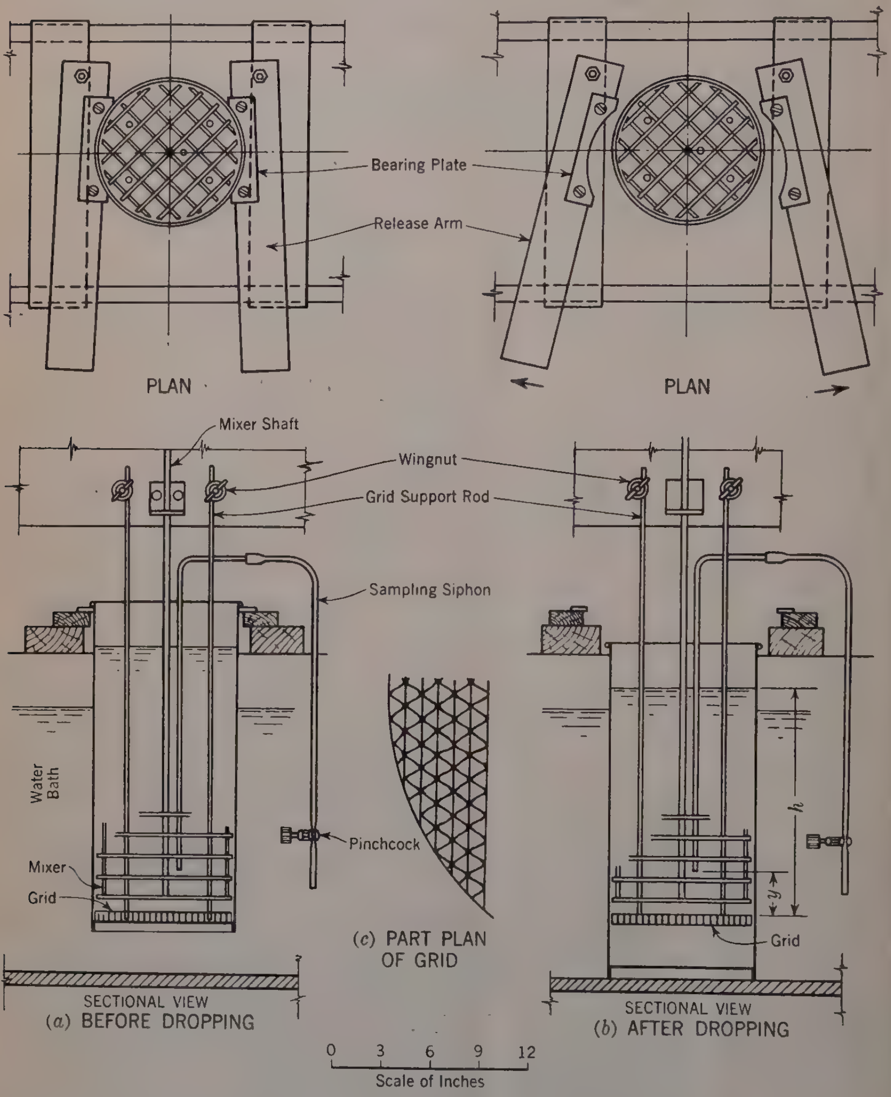

Fig. 9.—Experimental Setup for Run IV

The experimental setup is shown diagrammatically in Fig. 10. The cylinder could be raised or lowered by turning the bolt at the right—one com-

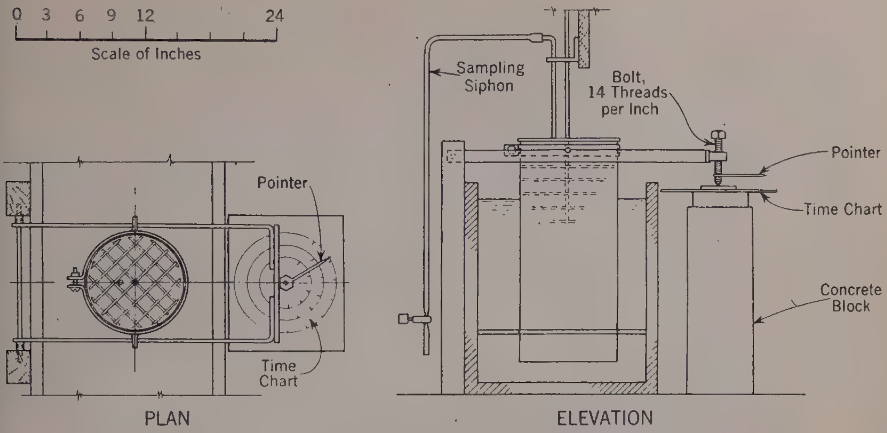

FIG. 10.—EXPERIMENTAL SETUP FOR RUN V

plete turn had the effect of raising or lowering the cylinder 0.825 mm. Since the position of the mixer remained unchanged, the space between the mixer and the bottom of the cylinder was decreased or increased by the same amount.

A total of 660 cu cm of the lucite was used, and for the initial condition the cylinder was set so that the bottom was 14.4 mm below the lowest position of

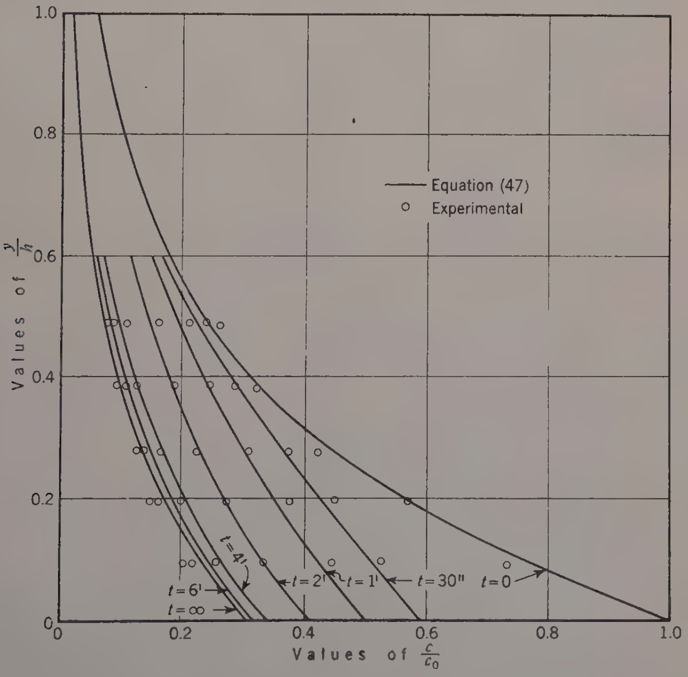

FIG. 11.—RUN V. CONCENTRATION CHANGES WITH CONSTANT RATE OF PICKUP

$\left(\frac{w}{\epsilon} = 0.0638 \text{ cm}^{-1}\text{; and } h = 45.2 \text{ cm}\right)$

the mixer. For the final condition the cylinder was lowered 6.60 mm, which was accomplished by making eight complete turns of the bolt. The measured concentrations for both of these conditions are shown in Fig. 8. Computations based on these curves show that the average depth of the bottom deposit was 9.4 mm in the initial condition and 14.1 mm in the final condition. Therefore, the distance between the surface of the bottom material and the lowest position of the mixer was 5.0 mm for the initial condition and 6.9 mm in the final condition.

To make the time study, it was necessary to impose upon the initial distribution a sudden rate of pickup equal to that in the final condition and to maintain this rate of pickup constant as the concentrations decreased. The results in Fig. 6(*a*) indicated that the rate of pickup depends upon the relative positions of the mixer and the surface of the bottom deposit, provided that the bottom is saturated. Therefore, to satisfy the pickup requirement, it was necessary to lower the cylinder suddenly until the distance between the lowest position of the mixer and the surface of the bottom deposit was equal to 6.9 mm, and then gradually to lower the cylinder so as to maintain this value as the suspended matter settled out.

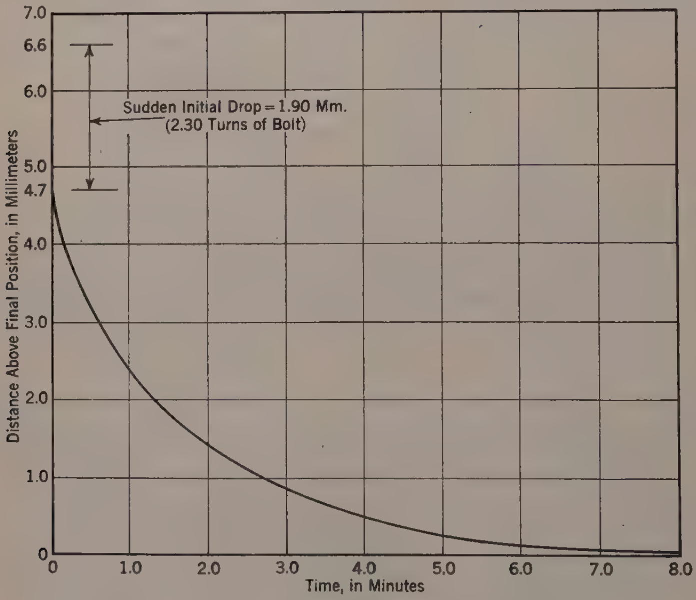

FIG. 12.—RATE OF DROP OF CYLINDER FOR RUN V

The sudden initial drop was 1.9 mm, which was equivalent to 2.3 turns of the bolt. From the theoretical concentration curve for any time after the initial drop (Fig. 11), the total amount of material deposited on the bottom up to that time could be computed, and hence the necessary total drop for the cylinder up to that time could be ascertained. By this means was prepared the curve shown in Fig. 12, which gives the necessary drop of the cylinder as a function of the time after the start. With the aid of Fig. 12, the time chart illustrated in Fig. 13 was prepared. In making the run, this chart was placed directly under the bolt and the bolt was turned so as to keep the pointer over the proper value of the time.

In the actual execution of the run, the services of two operators were required—one to turn the bolt and the other to take the samples. In turning the bolt, 3 or 4 sec elapsed before the initial turns could be made and the pointer brought up to its proper position on the chart. Once the proper position was reached, the pointer could be kept to within 2 or 3 sec of the proper value. After the 6-min sample had been taken, the pointer was turned to the final position and about 10 min were allowed to elapse before the final sample was drawn. After a complete series of samples had been taken for one value of y, the position of the siphon was changed and the entire procedure repeated. The results for this run are shown in Figs. 8 and 11.

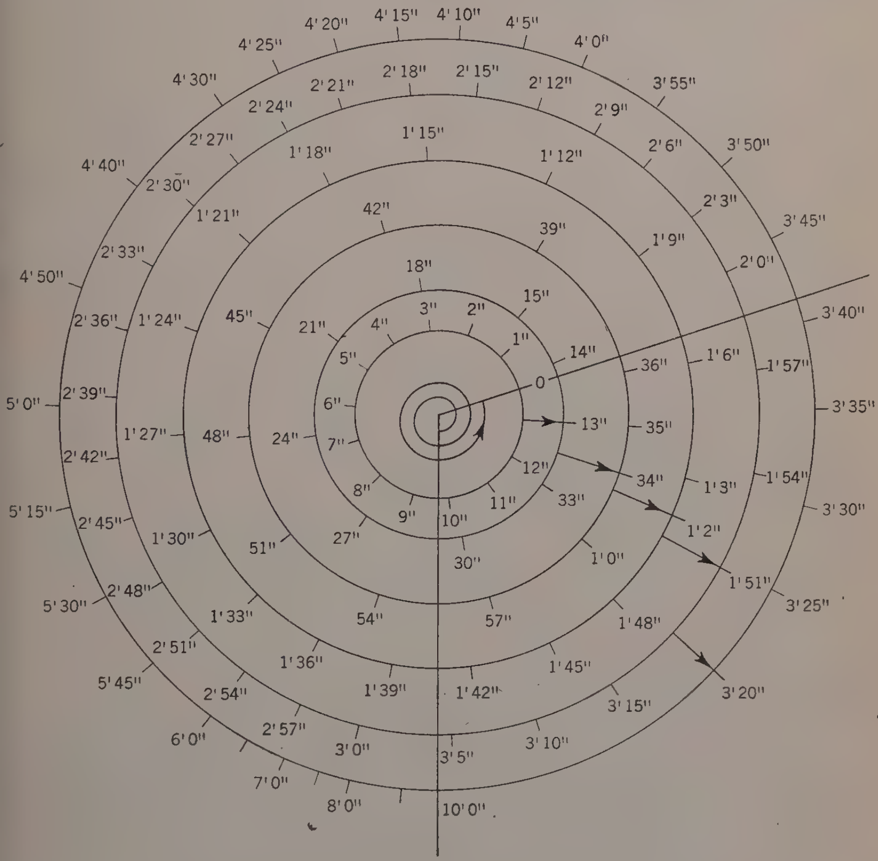

FIG. 13.—VIEW OF TIME CHART USED IN RUN V

Discussion of the Results

The results show that the absolute magnitudes of the concentrations in suspension depend upon the quantity of material on the bottom and the hydraulic conditions at the bottom. The factors affecting the latter are the speed of the mixer and its position relative to the surface of the bottom deposit. The greater the mixer speed and the closer it is to the bed material, the greater are the concentrations of material in suspension.

The results demonstrate that, for a given speed of the mixer and a given position of the mixer relative to the bottom deposit, there is a definite concentration at any value of y, provided that the bottom is saturated. Fig. 6(a) shows conditions in which the depth of the bottom deposit varies and Fig. 6(b) shows conditions in which the depth of the fluid varies. In all cases, the concentration curves coincide when the bottom is saturated and the speed of the mixer and its position relative to the surface of the bed material are fixed.

In all of the experiments it was found that the final equilibrium concentrations were independent of the manner in which the lucite was added. In some cases all of the material had settled to the bottom before the mixer was set in motion; in other cases the mixer was already in motion as the material was added.

All of the results show that the absolute magnitude of the equilibrium concentration at a given elevation above the surface of the bed material is independent of the total amount of material in suspension and depends only upon the hydraulic conditions at the bed, provided that the bottom is saturated with the material. At the equilibrium condition, the rate of pickup must equal the rate of deposit, $w\,c$. Therefore, the results show that the rate of pickup of a given material is independent of the total amount of material in suspension and depends only upon the hydraulic conditions at the surface of the bed material. It seems logical to assume that the rate of pickup of a given material in a stream would also depend only upon the hydraulic conditions at the bed. Of course, in a natural stream the distribution of the particle sizes in the bed material would have to be taken into account.

The results of the time studies shown in Figs. 7 and 11 provide definite verification of the validity of Eq. 41. The trends of the experimental results follow those of the theoretical results quite closely. The validity of the assumptions regarding the boundary conditions as well as the validity of the time factor involved in the equation is demonstrated.

In Fig. 7 the plotted points lie consistently to the right of the theoretical curves, although in no case is the error any larger than 5% of the original bottom concentration. These results show that the rate at which the material settled was slightly less than the theoretical rate. Two factors are offered as an explanation of this. At the start of the run, when the cylinder was dropped, the effect on the suspension was the same as if the mixer had been suddenly lifted upward. This may have had the effect of transporting material upward so as to result in an original concentration at every value of y which was slightly greater than what was assumed. Probably most important, however, was the fact that the pickup was not absolutely zero as the material settled out, since the grid provided some surface on to which the particles could fall and from which they could be picked up again.

The results in Fig. 7 show quite conclusively that the concentration gradient at the surface is proportional to the magnitude of the surface concentration and that the gradient at the bottom is zero when there is no pickup. The results in Fig. 11 demonstrate that the concentration gradient at the bottom is constant when the pickup is constant. Verification of the formulas that express boundary conditions I and II at the surface and at the bottom is thereby obtained.

CONCLUSIONS

The principal conclusions that can be deduced from the experimental results may be summarized as follows:

1. Further evidence of the validity of the logarithmic law for the vertical distribution of sediment at the equilibrium condition is provided;
2. The equilibrium concentrations are independent of the method by which the material is added to the suspension;
3. Other things being equal, the equilibrium concentrations depend upon the amount of material available on the bottom, up to the point of saturation;
4. When the bottom is saturated with a given material, the equilibrium concentrations depend upon the hydraulic conditions at the bed;
5. The equilibrium concentration at any elevation above the bed is independent of the depth of the fluid provided that the bottom is saturated;
6. The rate of pickup for unsteady conditions is independent of the total amount in suspension and depends only upon the hydraulic conditions at the bed and the characteristics of the bed material, provided that the bottom is saturated with the material; and
7. The one-dimensional case of the theory of turbulent sedimentation for discrete particles is verified.

ACKNOWLEDGMENTS

This paper was condensed from a thesis presented by the writer in partial fulfilment of the requirements for the degree of Doctor of Science in Sanitary Engineering from the Massachusetts Institute of Technology, Cambridge, Mass., in October, 1941. Copies of this thesis are on file at Massachusetts Institute of Technology and the Engineering Societies Library.[2a] The experimental work was done in the Institute's Sanitary Engineering Laboratory. The writer gratefully acknowledges his indebtedness to the members of the staff of the Department of Civil and Sanitary Engineering for their interest and help throughout the work.

Special acknowledgment is due to Professor Camp, under whose direction the study was made, for his advice and encouragement during the study. P. C. Stein, Assoc. M. Am. Soc. C. E., gave many valuable suggestions and assisted in the preparation of some of the figures, and Prof. G. P. Wadsworth of the Department of Mathematics advised the writer concerning some of the mathematical aspects of the study.

[2a] Engineering Societies Library, 29 West 39th Street, New York, N. Y.

APPENDIX 1

Bibliography

(1) **"On Sedimentation,"** by Allen Hazen, *Transactions*, Am. Soc. C. E., Vol. LIII (1904), p. 45.

(2) **"A Study of the Rational Design of Settling Tanks,"** by Thomas R. Camp, *Sewage Works Journal*, Vol. VIII, No. 5, September, 1936, p. 742.

(3) **"Bericht über Untersuchungen zur ausgebildeten Turbulenz,"** by L. Prandtl, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 5, No. 2, 1925, p. 136.

(4) **"Some Aspects of the Turbulence Problem,"** by T. von Kármán, *Proceedings*, Fourth International Cong. for Applied Mechanics, Cambridge, Mass., 1934.

(5) **"Turbulence in Open Channel Flow,"** by A. A. Kalinske and J. M. Robertson, *Engineering News-Record*, Vol. 126, No. 15, April 10, 1941, p. 53.

(6) **"Experiments on the Mechanics of Sediment Suspension,"** by H. Rouse, *Proceedings*, Fifth International Cong. for Applied Mechanics, 1938.

(7) **"Experiments on the Transportation of Sediments in Suspension,"** by V. A. Vanoni, Am. Geophysical Union, 1941.

(8) **"Der Massenaustausch in freier Luft und verwandte Erscheinungen,"** by W. Schmidt, *Die Wasserwirtschaft*, No. 5–6, 1932 (Mitteilung 10, Inst. für Wasserbau, Tech. Hochschule, Berlin).

(9) **"Review of the Theory of Turbulent Flow and Its Relation to Sediment-Transportation,"** by Morrough P. O'Brien, *Transactions*, Am. Geophysical Union, Section of Hydrology, 1933, p. 487.

(10) **"Laws of Turbulent Flow in Open Channels,"** by Garbis H. Keulegan, *Research Paper RP1151*, *Journal of Research*, National Bureau of Standards, Vol. 21, 1938, p. 707.

(11) **"Velocity Distribution in Open Channels,"** by Vito A. Vanoni, *Civil Engineering*, June, 1941, p. 356.

(12) **"The Relation of Suspended to Bed Material in Rivers,"** by E. W. Lane and A. A. Kalinske, *Transactions*, Am. Geophysical Union, Section of Hydrology, 1939, p. 637.

(13) **"Laws of Transportation of Sediment by Streams: Suspended Load,"** by H. Rouse, *Reprint No. 21*, State Univ. of Iowa, Iowa City, Iowa, Reprints in Engineering, 1939.

(14) **"Suspended-Material Transportation under Non-Equilibrium Conditions,"** by A. A. Kalinske, *Transactions*, Am. Geophysical Union, Section of Hydrology, 1940, p. 613.

(15) **"Introduction to the Mathematical Theory of the Conduction of Heat in Solids,"** by H. S. Carslaw, Macmillan, London, 1921.

(16) **"Neuere Methoden und Ergebnisse in der Hydromechanik,"** by C. W. Oseen, Akademische Verlagsgesellschaft, Leipzig, 1927.

APPENDIX 2

NOTATION

The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Hydraulics,[3] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942:

A = concentration at $y = 0$, $t = \infty$;
a = reference elevation for suspended-load function;
B = constant used in a function describing the initial distribution of suspended matter;
C = constant: C_1, C_2, C_3 = arbitrary constants; C_n = arbitrary constant in Eq. 32*b*;
c = concentration of sediment in suspension: c_0 = concentration at $t = 0$, $y = 0$;
e = base of Naperian logarithms;
f = Weisbach-Darcy friction factor;
g = acceleration of gravity;
H_n = a function of h defined by Eq. 39*b*;
h = total depth of fluid;
k = a constant defined by Eqs. 21 and 24; when specifically defined, it is also used as a constant in the formula of a parabola (Eqs. 15);
l = mixing length;
n = any integer;
S = slope of energy gradient;
s = distance in any direction (see Eq. 2);
T = function of t;
t = time of settling;
V = mean velocity at a point;
$\overline{V}$ = mean velocity for the entire cross section;
v = instantaneous velocity at a point;
$\bar{v}$ = mean velocity at a point;
v' = fluctuation velocity at a point: $\bar{v}'$ = temporal mean value of the fluctuation velocity; $|\bar{v}'|$ = temporal mean value of the absolute magnitude of the fluctuation velocity;
W = total amount of sediment held in suspension per unit horizontal area;
w = settling velocity of the particles;
x = horizontal distance in the direction of flow;
Y = a function of y: Y_n = a function of y defined by Eq. 34;
y = elevation above surface of bed material;
z = exponent in a suspended-load function;
α = constant defined by Eq. 31;

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[3] ASA—Z10.2—1942.

β = proportionality factor;
ϵ = kinematic turbulence coefficient = $\beta|\bar{v}'|l$;
η = a function of $h\alpha$ defined in Eq. 31;
κ = von Kármán universal velocity distribution constant;
ρ = mass density of the fluid;
τ = intensity of fluid shear: τ_0 = intensity of fluid shear at $y = 0$.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

DEVELOPMENT OF THE CHICAGO TYPE BASCULE BRIDGE

By Donald N. Becker,[1] M. Am. Soc. C. E.

Synopsis

On the background of a description of the waterway system that serves the navigation needs of Chicago, Ill., the author describes the development of a bridge type that serves the peculiar local requirements. The evolution of the movable type of bridge is traced through several phases, including the early "jackknife," the vertical lift, the rolling lift, and, finally, the various developments of the bascule type.

Introduction

The Chicago River (see Fig. 1) lies along the north side of the central business district, branching just west of it, with the North Branch traversing the north side of the city in a more or less northwesterly direction and the south branch traversing the south side in a southwesterly direction. In the extreme south end of the city is the Calumet River system. In Fig. 1, the various bridges mentioned herein are given numbers that are identified as follows: 1. West Cortland Street (Clybourn Place); 2. West Division Street (River); 3. West North Avenue; 4. North Halsted Street (Canal); 5. West Kinzie Street; 6. West Washington Street; 7. West Grand Avenue (Indiana Street); 8. East 92d Street; 9. West 35th Street; 10. West Chicago Avenue; 11. West Webster Avenue; 12. West Belmont Avenue; 13. West Monroe Street; 14. North Franklin-Orleans Street; 15. West Adams Street; 16. East 106th Street; 17. West Roosevelt Road (Twelfth Street); 18. North Damen Avenue (North Robey Street); 19. South Damen Avenue (South Robey Street); 20. North Wabash Avenue; 21. West Ogden Avenue (Canal); 22. West Ogden Avenue (River); 23. Outer Drive (River); 24. Outer Drive (Canal); 25. South Ashland Avenue; 26. North State Street; 27. Weed Street Crossing of Chicago River; 28. Canal Street Crossing of Chicago River; 29. South Halsted Crossing of Chicago River; 30. Van Buren Street Bridge; 31. Metropolitan Elevated Railroad Bridge; 32. North Halsted Street crossing of North Branch; 33. La Salle

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Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **July 1, 1943.**

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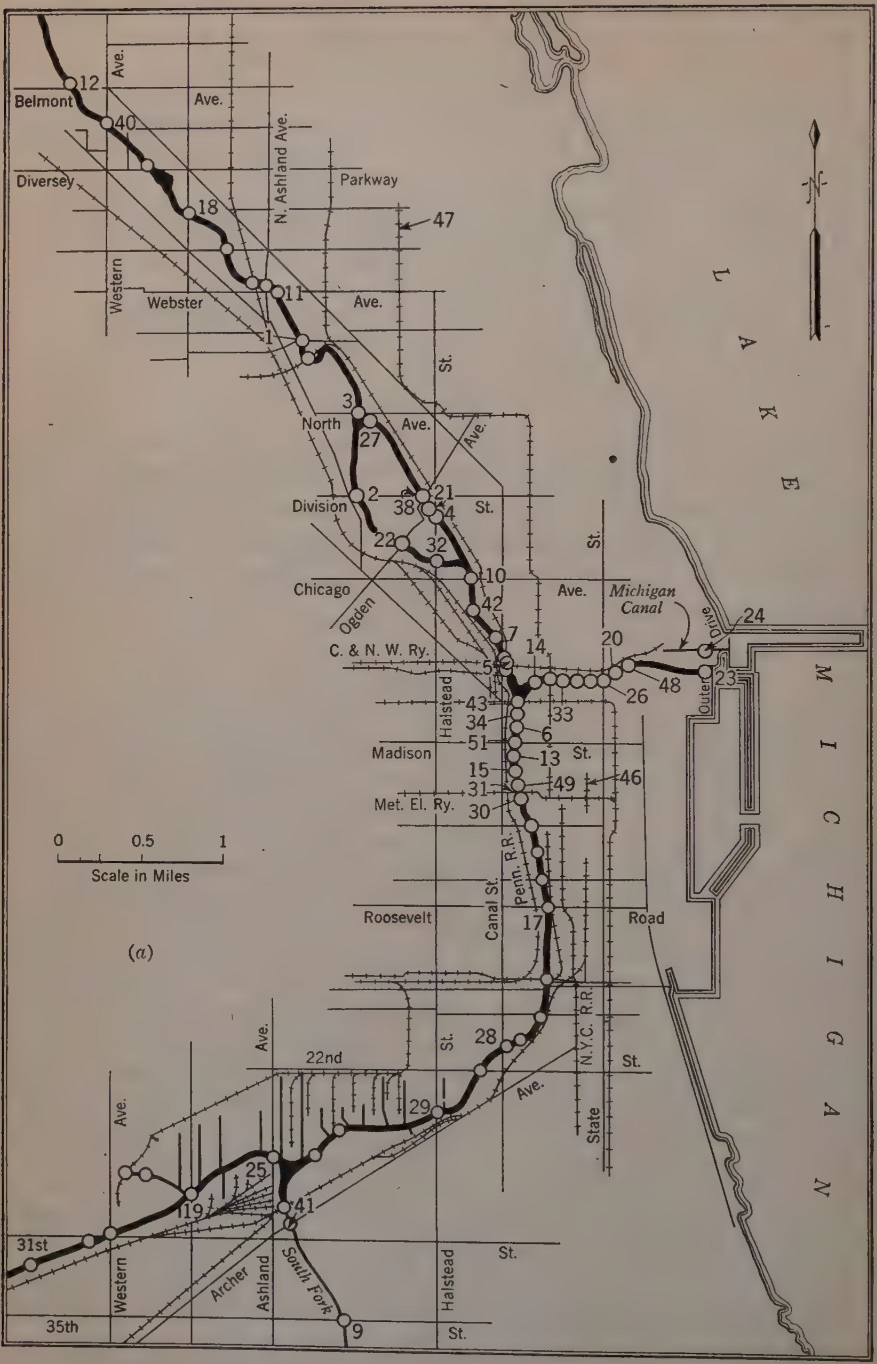
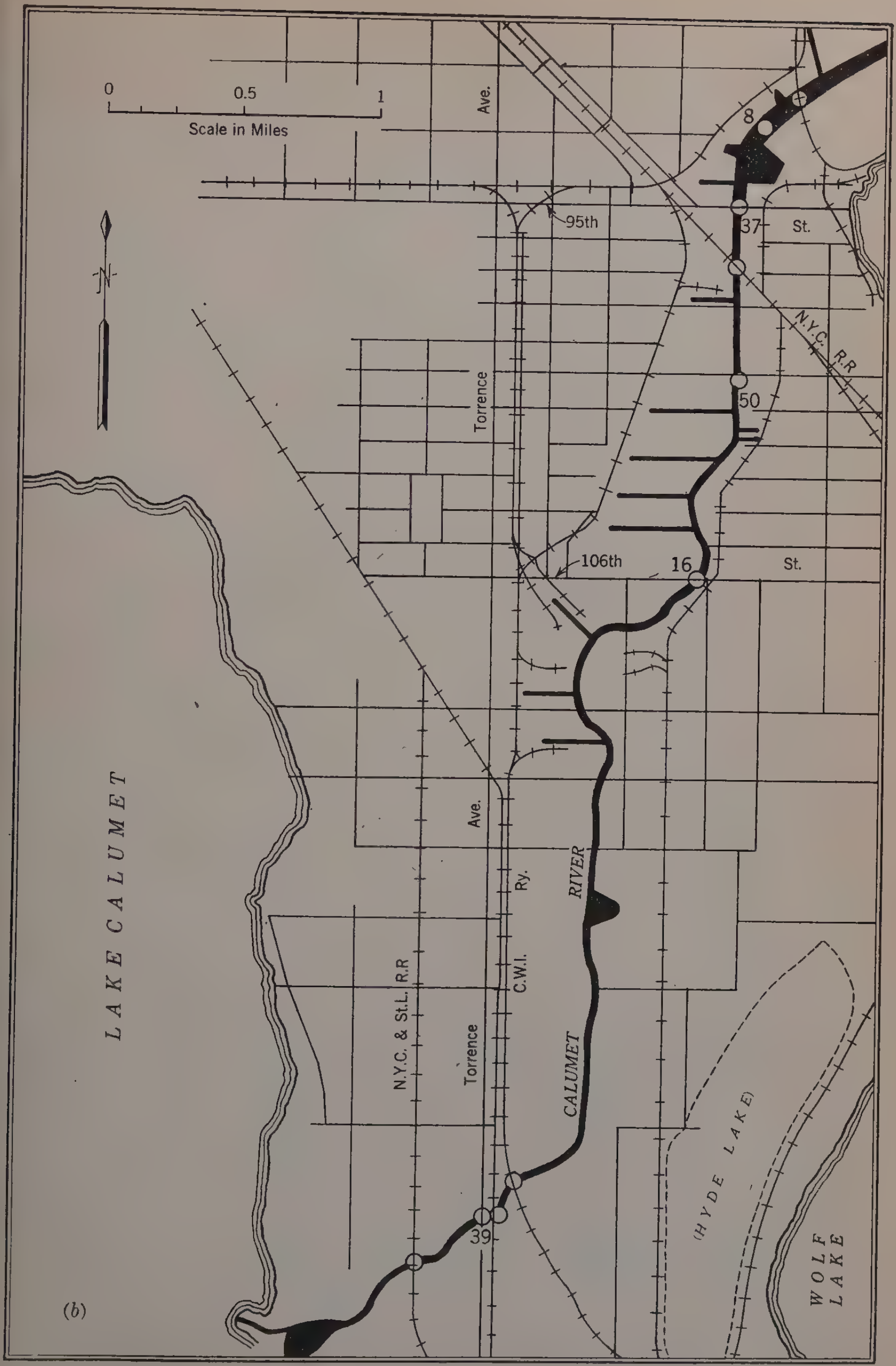


FIG. 1.—DRAINAGE



SYSTEM, CHICAGO, ILL.

Street Tunnel; 34. Washington Street Tunnel; 35. Clark Street Bridge; 36. Wells Street Bridge; 37. East 95th Street and Calumet River; 38. West Division Street and North Branch Canal; 39. Cortland Street and North Branch; 40. North Western Avenue and North Branch; 41. West Archer Avenue and South Fork of South Branch; 42. West Erie Street and North Branch; 43. West Lake Street; 44. Lake Street Elevated Railway; 45. West Lake Street and Elevated Railway; 46. Dearborn Street Subway System; 47. Northwestern Elevated Railway; 48. Michigan Avenue; 49. West Jackson Boulevard and South Branch; 50. East 100th Street and Calumet River; 51. Madison Street; and 52. South Torrence Avenue and Calumet River.

Before civilization reached Chicago these rivers were relatively shallow streams, navigable only in the sense that canoes and shallow draft vessels could enter them from Lake Michigan. With the development of the city, both of these rivers were deepened and widened so that they are navigable for considerable distances for the largest lake-going vessels. They are generally equipped with dockwalls providing channels varying in width from 100 ft to 250 ft.

When the early settlers traveling overland first reached the Chicago River, they established ferries, but these soon proved inadequate for the traffic, and as early as 1833 a crude float bridge, manipulated by cables, was installed. In 1834, a wooden drawbridge similar to a double moat-bridge was built across the river, only to be torn down five years later because it was an obstruction to navigation.

By 1840, however, the land traffic was so heavy that another bridge was built over the river. This time it was a pontoon type that could be pulled open when needed for river traffic. Several of this type were built in the next few years. In 1854, the first pivot swing bridge with wooden trusses was built across the river; and in 1856 the first all-iron bridge was built. By the time of the great Chicago fire (1871), there were twenty-seven movable bridges costing well over a half million dollars. The great fire destroyed eight of these. By 1890, the number of bridges had increased to forty eight, some through the annexation of adjacent towns to the city.

During this period many disadvantages of the swing-type bridge presented themselves, so that thought was directed to the new types that were being developed in the minds of embryo inventors. In 1890 a so-called "jackknife" bridge, built under patents issued to Capt. W. Harmon, was installed at Weed Street, the action of each half of the bridge resembling that of a loosely-hinged jackknife as it lowered from the vertical position. It was expected that, if experience showed that this type of bridge could be operated safely and economically, it would be preferable in most cases to the swing bridge. It was also thought that the raised floor, in the open position, would solve the street barrier problem.

In 1892 so much objection was raised to the swing bridge, built only two years previously at Canal Street at a bend in the river, that it was ordered removed and was replaced at a cost of $46,844.90 with a Harmon-type of jackknife bridge (Fig. 2) providing an 80-ft draw. From the beginning it was troublesome due to light construction and poor execution.

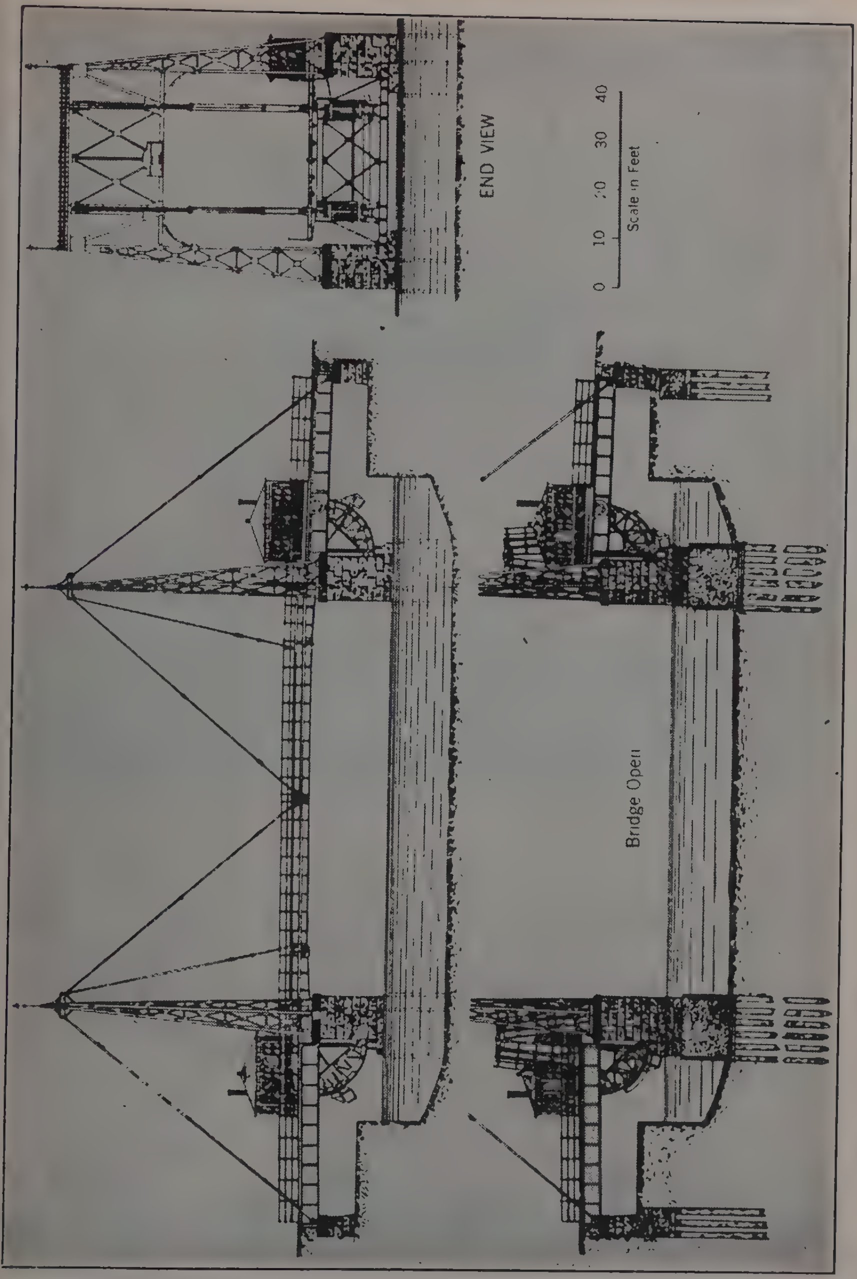


FIG. 2.—SOUTH CANAL STREET FOLDING BRIDGE OVER THE SOUTH BRANCH OF THE CHICAGO RIVER, 1892

In 1892 when it became necessary to replace the old swing bridge at South Halsted Street, the navigation interests and the Drainage Board (later The Sanitary District of Chicago) objected so effectively that the War Department refused to issue a permit for a swing bridge. After considering the Harmon jackknife and the Waddell lift bridge, the city decided in favor of the latter, which was completed and opened to traffic on March 22, 1894 (see Fig. 3). It consisted of a steel truss span, 130 ft long by 58 ft wide, with a single roadway 40 ft wide and two sidewalks 8 ft wide. Its weight (280 tons) was lifted vertically between two towers that were braced by a truss between the top of the towers at a height of 155 ft above the river level. The lift span was suspended by wire ropes that passed over sheaves at the tops of the towers and were connected to cast-iron counterweights within the towers. The bridge was operated by steam hoisting machinery located on top of the lift span at its center, which operated cables attached to the tops and bottoms of the towers. The total cost of the bridge was $242,880.

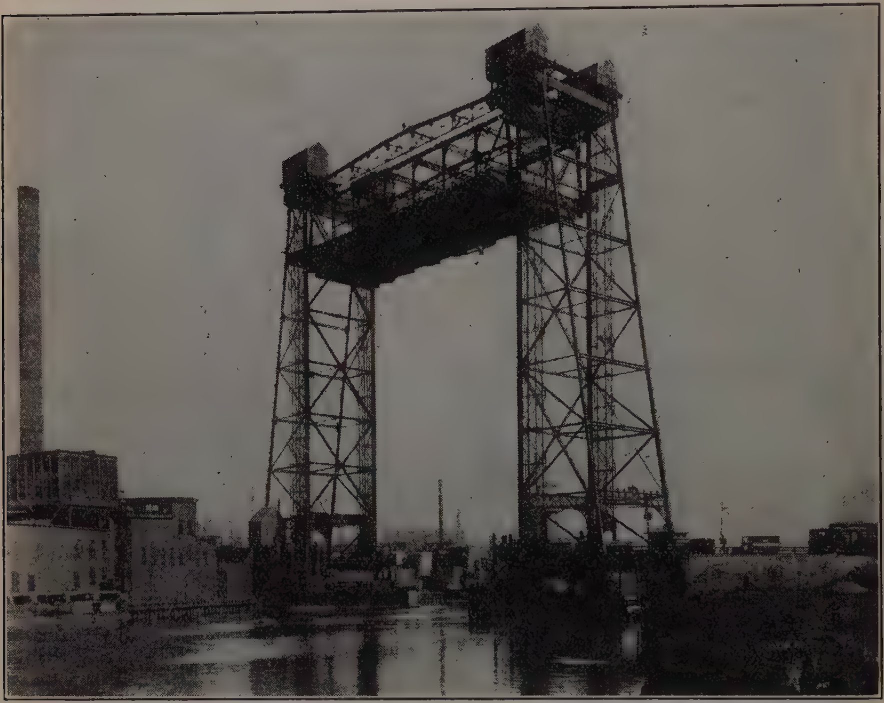


Fig. 3.—South Halsted Street Bridge, Facing Northeast Across the Chicago River, 1894

In 1894, when it was necessary to replace the Van Buren Street bridge, and the Metropolitan Elevated Railroad Company wished to build a bridge across the river a short distance north of it, conditions being such as to prohibit the use of swing bridges, a new type was adopted (see Fig. 4) for both locations. This bridge was hailed as the embodiment of an eleventh century idea brought up to date, as the combination of the bascule (which was the most common type of medieval moat bridge) and the latest electric and pneumatic appliances. It was equipped with air brakes and two 50-hp electric motors on each leaf. It cost $169,700.

In 1895 this gave the city three highly different types of bridges for comparison, the Harmon "jackknife," the Waddell "vertical lift," and the Scherzer "rolling lift." In 1896 a second Scherzer bridge was built at North Halsted Street over the North Branch.

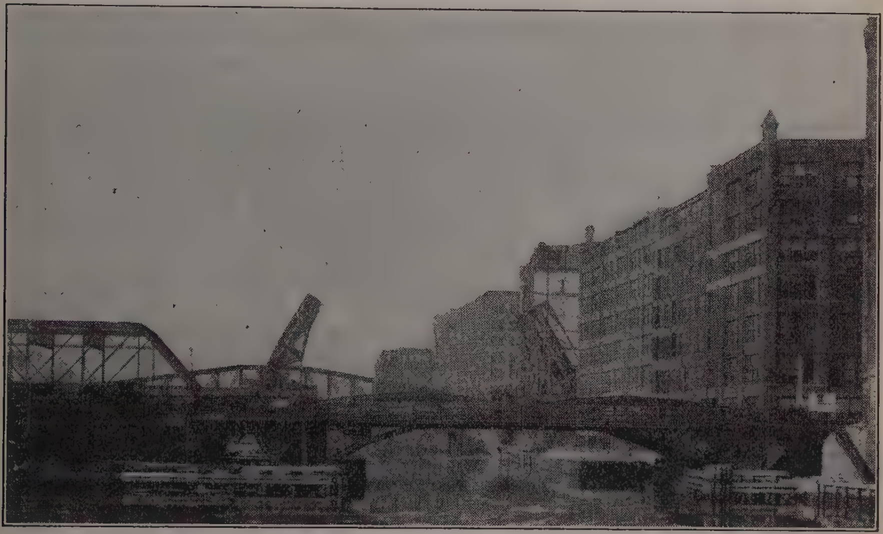


FIG. 4.—SCHERZER TYPE OF ROLLING LIFT BRIDGE (1894) ACROSS THE CHICAGO RIVER AT WEST VAN BUREN STREET

From the Annual Report of the Department of Public Works of the City of Chicago for 1898 the following is quoted to indicate the conditions then:

> "There is no portion of the City's service that is in such need of immediate attention as the bridges and viaducts of the City, and it is to be regretted that appropriations of sufficient amount cannot be secured to make proper and necessary repairs. Many of the structures are old and dilapidated and have reached that stage of decay where wise expenditure of money cannot be made to repair them, and they should be replaced by new structures. All the bridges to be constructed hereafter should be constructed on such a plan that center piers will not be necessary. The practice of placing obstructions of any character in the Chicago River should be discontinued, with a view to accommodate navigation and to conform to the policy of the statutes governing the construction of the Sanitary Drainage Canal. While the importance of the Chicago River appears to be appreciated by the citizens in the discussion of the development of Chicago, no practical steps have been taken by the City to preserve it as a harbor for modern vessels. A reference to the tables submitted by the City Engineer in his report must convince anyone that the greater facilities offered in the Calumet River are diverting a large portion of the business from the Chicago River, and while the Calumet interest should be con-

served and promoted as a Chicago interest, due consideration should be given to the Chicago River in our plan for future development of Chicago."

Navigation in the Chicago River was hampered by artificial restrictions, such as center piers of swing bridges, protruding dockwalls where the owners had thought no further than how much land they could create for their own use, and lastly by the shallow clearance over La Salle Street and Washington Street tunnels where only 17 ft of water was available. The largest vessels plying the Great Lakes were 432 ft long by 48 ft beam, with a draft of about 16 ft; but due to the tortuous channel in the river they could not enter the Chicago River farther than the Clark Street bridge. To proceed further it would be necessary to enter the north draw at Clark Street, then head diagonally across the river toward the south draw at Wells Street so as to cross over the LaSalle Street tunnel at the middle of the river, which was the only point deep enough to permit passage. If the vessel did squeeze through in this manner it could not enter the North or South Branch due to center obstructions and bends in the dockwalls.

The law creating the Sanitary District provided for a canal with a discharge capacity of 10,000 cu ft per sec. The District embarked on a policy of constructing a 200-ft channel in the South Branch with a center depth of 26 ft and a cross section of 4,800 sq ft so as to limit the current to about 1¼ miles per hr. Property was condemned and the docks were moved back. As the city was in no position to replace bridges to accommodate this increased flow, especially where bridges were in fair to good condition, an agreement was made whereby the District would rebuild certain structures and turn them over to the city. Under this arrangement ten bridges were completed between 1902 and 1907—nine of these were of the Scherzer rolling lift type, and one of the Page type.

Between 1896 and 1900 the city finances were in such a state that not only was no money available for new bridges, but even repair funds were greatly curtailed.

Birth of the Chicago Type Bascule

In 1900 the situation became so acute that $1,250,000 was authorized for eight bridges, $850,000 to be available in 1900. The next problem was to determine the type of structure. The first Harmon bridge at Weed Street was continually out of order, due to the number of joints, and was expensive to maintain. This was true also of the Canal Street bridge.

The vertical lift bridge at South Halsted Street, in spite of its high cost, was proving quite costly in operation and maintenance each year. The rolling lift bridge, although admittedly more satisfactory than the folding jackknife and the vertical lift bridge, had the very important disadvantage that the center of gravity traveled back and forth as the bridge opened and closed, thus requiring a solid foundation.

In 1899 the city's Bridge Division made a critical analysis of the literature on movable bridges in the United States and Europe, with a view to selecting a type of bridge most suitable to the Chicago River and its branches. The results were put in the form of a report giving the advantages and disadvantages of the various movable types in use. The type known as the trunnion bascule was considered to be the one that could meet Chicago requirements most fully and satisfactorily.

Three designs were prepared by the city, differing in appearance, method of mounting, etc., but all involving the principle of revolving on a fixed trunnion. The three designs were then submitted to a Board of Consulting Engineers, consisting of E. L. Cooley, the late Ralph Modjeski, M. Am. Soc. C. E., and the late Byron B. Carter. The Board selected and recommended the design designated as No. 3 with some modifications.

The design was described in the 1900 report of the Department of Public Works as a fixed-center, double-leaf, counterbalanced bascule bridge. Each leaf had three through trusses operated by racks on their curved tail ends, the tail ends descending into a pocket or tail pit in the abutments when the bridge was open. The total width of the structure was 60 ft, the trusses being 21 ft, center to center, and the sidewalks being carried by 9-ft cantilever brackets. The pivot bearings were 4 ft back from the center lines of the river piers and were carried by box girders spanning the tail pits in the abutments. The approaches were carried by I-beams, supported by cross girders resting on the box girders and extending from the river piers to concrete abutments independent of the abutments of the moving span. The bridge was designed to turn through a maximum angle of 76° 58′, which required a travel of 35 ft on the pitch line of the rack. The break in the roadway was on the river side of the pivot.

The machinery for operating each leaf was under the approach roadway, each leaf being operated by means of a pinion gearing with a rack on the curved tail end of each truss. The teeth on this rack had openings between them, preventing the clogging of the rack by dirt or ice. The teeth of the operating pinion dislodged any accumulation that would then fall through the rack. Along the top of the abutment a shaft carried three pinions and had two sets of driving gear, each set of driving gear being driven by an electric motor of 38 hp. On each motor-driven shaft was a wheel for a band brake, operated from the bridge house by a vertical hand lever with a latch handle. Besides these hand brakes there was a worm-resistance brake at the middle of the rack pinion shaft and a hand-brake wheel was fastened to the shafts of the worm. To prevent resistance or friction in the worm brake when not needed, the worm was revolved by a light continuous hemp rope that passed over grooved pulleys on the motor-driven shafts and the worm shaft. The band brake was applied to the worm shaft to introduce the resistance, thus checking the motion of the worm and consequently of the gearing by which it was connected to the main shaft.

The machinery was designed so that the opening of the bridge, from the moment it was closed to traffic to the moment when the leaves reached their highest position, would not require more than 1 min in calm weather or $2\frac{1}{2}$ min with a 70-mile wind blowing in a direction unfavorable to the operation of the bridge. Pneumatic buffers arrested the motion of the leaf at each end of its travel.

When the bridge was closed, the center of gravity of each leaf was on the river side of the pivot, and as the movable part of the roadway did not extend back of the pivot it was only necessary to lock the leaf against further downward movement. This was done in connection with the bridge-closing buffers, by allowing the leaf to pull the lower buffer pistons against the cylinder ends, the cylinders being anchored to the masonry.

When the bridge was closed, the action of the buffer was as follows: As the leaf approached the end of its closing movement, a hooked lug at the lower end of the arched back of each truss picked up a 6½-in. pin, connected by eyebars to a combined cross head and piston. This pin forced the piston upward in an inverted cylinder, meeting the increasing resistance of the compressing air, which absorbed the energy of the closing leaf. When the pressure reached 350 lb per sq in., the air escaped through a relief valve and allowed the piston to rest against the cylinder head above it.

As the leaf began to open, a second lug near the aforementioned hooked lug returned the buffer and connections, by means of a hooked lever, to position in readiness for closing.

The action of the upper buffer in arresting the leaf at the end of the opening movement was similar to that of the lower buffer, the difference being that the upper buffer was operated through an intermediate lever pivoted to special pairs of stringers of the approach roadway, which stringers also supported the upper buffer cylinders. The rear end of the lever acted on the piston through a short link. The front end of the lever was operated in essentially the same manner as the pin of the lower buffer, by lugs in the upper end of the arched back of each truss.

Quoting from the Report of the Board of Consultants:

> "The principle of the trunnion bearing meets with our approval as a very simple solution of the problem, the chief advantages being the constant point and direction of application of the load on the foundation, whether the bridge is in motion or stationary, and the reduction of the number of moving parts to the minimum. The first advantage is of no great consequence if the piers are placed on an unyielding foundation, but with such foundations as can be obtained in Chicago at the majority of bridge sites, it is of great importance. The trunnion type of bridge used in these designs is an old and tried device, and is not covered by patents."

The consulting engineers called attention to the fact that the bascule design permits the center of gravity of the moving bridge to be placed in or near the trunnion axis. Placing the center of gravity a short distance from the axis of the trunnion toward the draw opening and the arrangement by which the fixed floor relieves the tail end from any possible live load, had the advantage of tending to hold the bascule firmly in position when closed, and so avoided the absolute necessity for heel locks. There was no tilting effect when a live load came on to the bridge.

The Board recommended a three-truss design, as had been prepared by the city, in preference to a two-truss design for the same width of floor, that the curved bottom lateral struts be omitted and a stiff lateral system used. Also, it was recommended that, when the bridge was raised, the floor beams be supported by angles running from the middle of each floor beam to the intersection of the lateral angles, and that the heel ends of the trusses be connected by a transverse brace, to add lateral stiffness to the racks and the counterweights.

The Consulting Board further recommended that adjustable resting blocks be placed in front of, and near, the trunnions so that when the draw was being closed, the load could be transferred from the trunnions to the resting blocks. This facilitated the lubrication and practically eliminated the starting friction of the trunnions. To meet all possible conditions of wind and incidental changes in the position of the center of gravity, pawls or heel locks were used with the resting blocks. They were capable of lifting the tail ends and holding them firmly against the anchorage. This arrangement was also claimed to facilitate the overhauling and adjustment of the trunnion bearings.

Air buffers were considered practical, provided they could be designed so that the pistons would not withdraw from the cylinders, and provided they were packed with cupped leather or other material, to prevent entrance of foreign matter or grit into the cylinders. Air was controlled in the cylinders by free entrance through check valves, and by allowing exit through adjustable openings, for buffing. Springs insured proper action of the pistons. Exceedingly strong racks on the heel of the draw, with through openings between the rack cogs, met with approval as they will not accumulate ice in cold weather. On those rare occasions when sleet may coat the racks with ice, it was considered that the shaft and pinions were strong enough to force the ice through the cog openings.

There was no practical difficulty in making the driving machinery strong enough to resist all probable strains, and, therefore, the engineers recommended the following general arrangement: The three pinions are mounted on one line of shafting, with specially strong bearings, being driven by two gears; these two gears are connected by a jackshaft and suitable pinions, with two gears mounted, and clutch-driven, on this jackshaft. The jackshaft gears are driven by two independent gearing trains to two motors. All these gears, shafts, and motors are arranged in a vertical or inclined train, so proportioned that all gears are between their respective shaft bearings, which were to have ample length and all were to be mounted, as far as possible, on one structure. The gearing could easily be made stronger and safer and more accessible for overhauling and repairs than that of any bridge then built in the city.

The worm-gear brake for emergency use had novel merits compared with the usual band brake on the motor shaft. If designed with large pitch and large angle of worm, running in an oil bath, the friction of the gears running light may not be large. By the ingenious application of a light rope belt (to revolve the worm), the friction due to the thrust of the worm was practically eliminated until the brake was applied. In the detail design, the desirability of friction brakes applied directly to the curved segments of the trusses can be developed. Finally, quoting the Report:

> "It is fair to state that the assumed wind loads against opening or closing the bridge may never prevail in the practical handling of the bridge, and may be regarded as so much additional security. The fact is that ships do not navigate when such winds prevail, and there would be

> no occasion to open the draw at such times. In the case of the Tower Bridge (London), the reserve power put in on account of extra wind loads was reported as 'never used' at the end of two years.
>
> "In general, our board is of the opinion that the flow of the river should be provided in the opening of the draw wherever practicable."

In the light of forty years of experience, the recommendations of this Board were almost prophetic, as so many of their recommendations have been adopted with the passage of years.

Early Examples of the Chicago Type

Meanwhile, the 95th Street swing bridge, over the Calumet River, had collapsed on August 17, 1899. This location was used as the site for a new bridge, the call for competitive bids to be opened on May 15, 1900. A similar call for bids was advertised to be opened on June 1, 1900, for a bridge at Division Street over the North Branch Canal.

The specifications were drawn very broad for the purpose of giving competitors as wide a scope as possible in their designs and selection of a new type of bridge. The only restrictions in regard to type were in both cases:

> "The bridge shall be a movable structure without center pier and shall leave a channel with a clear opening of 120 ft. at 95th Street and 80 ft. at Division Street. The bridge shall have two roadways 18 ft. in the clear, or one roadway 38 ft. in the clear between the wheel guards and two sidewalks 8 ft. in the clear, and shall have an overhead clearance of 16 ft. There must be an underbridge clearance of 16½ ft. above Chicago Datum for a space of at least 40 ft. at right angles to the center of the channel. The general appearance of the bridge shall be as graceful and attractive as the nature of the structure will permit."

Five designs were submitted by outside bidders, some of these bidders having two or more alternate designs differing in matters of detail. With one exception, the designs were for bridges revolving on fixed bearings.[2] The contract was awarded to a bidder following one of the city's designs.

Contracts for both 95th Street and Division Street bridges were awarded so that work was started in July, 1900. Meanwhile the city's design No. 3 was modified and arranged for a bridge at Clybourn Place (later Cortland Street) over the North Branch and contracts were awarded in the late fall. These plans embodied many of the alterations suggested by the Board of Consulting Engineers, especially, the use of a full tail pit instead of separate wells for each truss. Due to construction difficulties at both 95th Street and Division Street, the Clybourn Place bridge (Fig. 5) was completed first and opened to traffic on May 24, 1902; the 95th Street bridge was not opened to traffic until April 1, 1903; and the Division Street bridge on February 1, 1903. In the construction of the 95th Street and Division Street bridges, the bottom of the abutment was lowered to the elevation of the river piers but the separate pockets for the counterweight ends of the trusses were retained. These were made of sheet steel covered on the outside with concrete and have been a frequent cause of repairs. The machinery was set on structural supports, and it has been difficult

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[2] *Engineering News*, January 10, 1901, p. 18.

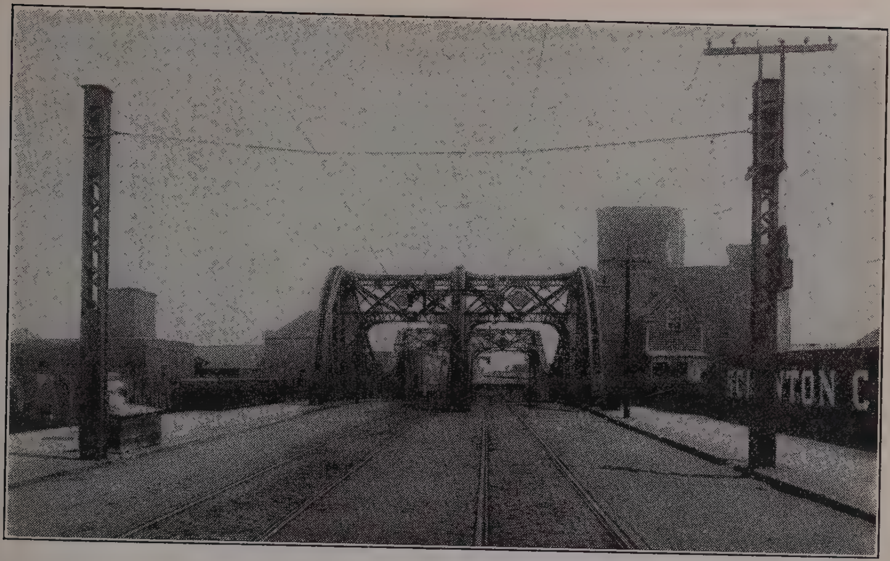
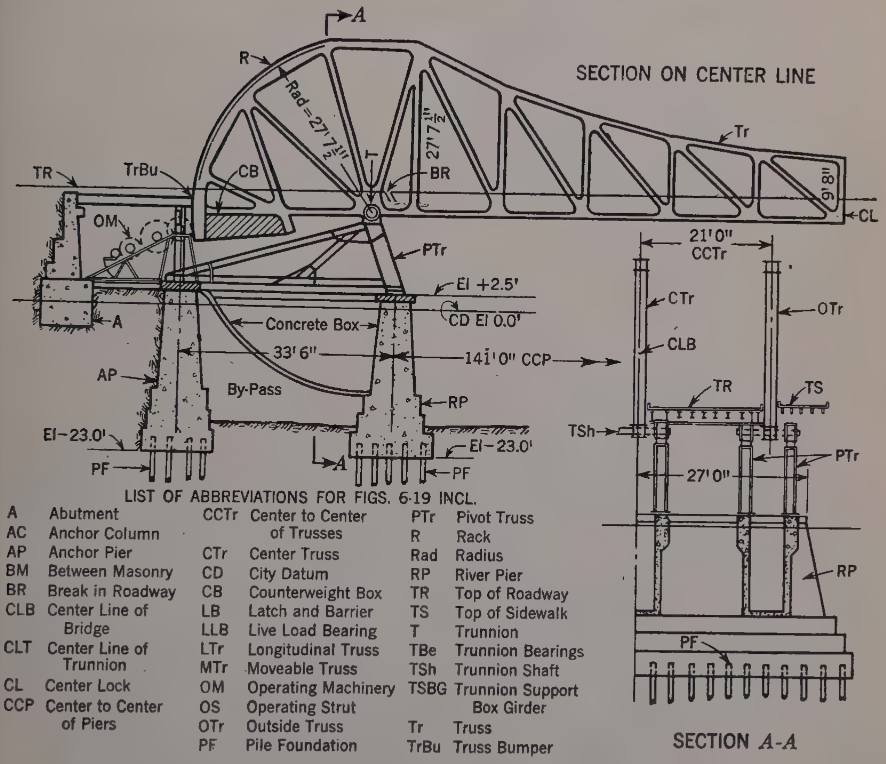


Fig. 5.—West Clybourn Place Bridge (1902)



LIST OF ABBREVIATIONS FOR FIGS. 6-19 INCL.

A	Abutment
AC	Anchor Column
AP	Anchor Pier
BM	Between Masonry
BR	Break in Roadway
CLB	Center Line of Bridge
CLT	Center Line of Trunnion
CL	Center Lock
CCP	Center to Center of Piers
CCTr	Center to Center of Trusses
CTr	Center Truss
CD	City Datum
CB	Counterweight Box
LB	Latch and Barrier
LLB	Live Load Bearing
LTr	Longitudinal Truss
MTr	Moveable Truss
OM	Operating Machinery
OS	Operating Strut
OTr	Outside Truss
PF	Pile Foundation
PTr	Pivot Truss
R	Rack
Rad	Radius
RP	River Pier
TR	Top of Roadway
TS	Top of Sidewalk
T	Trunnion
TBe	Trunnion Bearings
TSh	Trunnion Shaft
TSBG	Trunnion Support Box Girder
Tr	Truss
TrBu	Truss Bumper

Fig. 6.—West Division Street Canal Crossing (1903)

to maintain alinement. Changes were also made within a year after completion of the bridges, in the braking system to make it more effective. Fig. 6 shows the Division Street bridge.

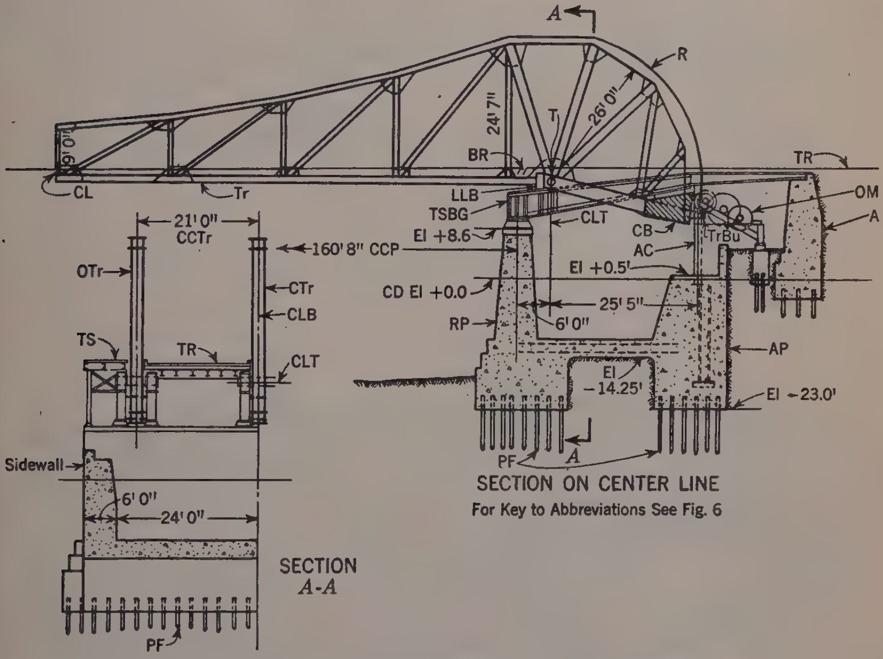
One of the earlier types of bridges was eliminated from further consideration when the Canal Street jackknife bridge was replaced by a Scherzer bridge in 1903, and the Weed Street bridge was finally removed in 1905 after having been out of service for several years. However, in 1903 a new type appeared with the completion of a Page bascule bridge at South Ashland Avenue.

In 1904 a new method of financing the construction of bridges was devised when the State Supreme Court ruled in favor of a bond issue for this purpose.

By the end of 1904 the city had five bridges of its design in use with further improvements in the last two, one at Division Street (Fig. 7) and one at North Western Avenue, both over the North Branch. In both of the two later bridges, instead of using shaped iron castings bolted to the trusses for counterweight as was done in the first three bridges, a structural box was framed between the tail ends of the trusses. These tail ends were sloped downward back of the trunnions, whose supports were changed to tapering box girders sloping upward at the back so as to pass over the counterweight box. The box was filled with the counterweights consisting of about 22% shaped castings and the remainder a mixture of pig iron, punchings, and cement grout to produce a material weighing about 375 lb per cu ft. The bottom of this box was floored with a grillage of old rails to carry the weight of the extremely heavy counterweight material to the main girders. There were many advantages in this type of construction. First, it allowed the center of gravity of the counterweight to be placed farther back from the trunnion, thus saving material. Second, it eliminated the necessity for the specially shaped castings with their greater unit cost. Third, the counterweight material did not have to be bolted piece by piece to the trusses with the danger, later, of dropping if bolts rusted away.

Four of these bridges were on pile foundations, but, at North Western Avenue bridge, piles could not be driven below −22 ft Chicago Datum because of a very hard indurated clay, which could safely sustain a load of 4,000 lb per sq ft. Accordingly, piles were omitted and the masonry rested directly on the soil. This bridge also had the tail pits especially waterproofed with a 4-in. layer of Portland cement mortar on both the inside and outside walls below water level. A distinct improvement in the machinery consisted of taking the motors from the two gear trains and combining them in a unit in back of the center trusses. Two sets of hand brakes were operated by hand levers in the operator's room. Also the pneumatic buffers at each end of the travel were eliminated and wooden bumper blocks were placed on the under side of the trunnion girder with which the top of the counterweight box came into contact in the closed position to limit the final position of the leaf. The decelerating speed of the trusses was entirely controlled by the machinery brakes. It had been found that the pneumatic buffers were too slow in operating and the pressure relief valves were too temperamental.

In 1903 plans were prepared for a double-leaf trunnion bascule bridge at Archer Avenue, over the South Fork of the South Branch, but later the federal government issued a revised permit with the clear channel reduced from 136 ft 8 in. to 107 ft. Investigation showed that a one-arm structure would be feasible, especially if the type of machinery worked out for North Western Avenue were adopted. By increasing the radius of the pin rack from 30 ft, used in the North Western Avenue bridge, to 36 ft, it was found that the machinery would have to be only a little heavier. The leaf was placed on the west side of the river with the idea that the prevailing west or southwest winds of the navigating season would assist rather than hinder the closing of the leaf. The trunnion supports were again changed, this time to an inverted triangular truss with the apex under the trunnion. This was necessary as the load was rather large for the tapered box girder type used at North Western Avenue. The bumper for the bridge, when down, was attached to the under side of this truss and, in addition, a bumper for the bridge in the open position consisted of a rubber block 7 in. by 8 in. in section attached to the top of a bracket on the fixed part of the steel, placed so that a bumper bracket on the top chord of the trusses came into contact with it when the leaf reached the fully open position. This type of bridge proved more economical than a double leaf. The only disadvantage was the open street on one end. At this end a counterweighted barrier was installed that raised above the roadway level when the bridge leaf was raised.



SECTION ON CENTER LINE

For Key to Abbreviations See Fig. 6

SECTION A-A

Fig. 7.—West Division Street River Crossing (1904)

In 1909 a single-leaf bridge was built at West Kinzie Street (Fig. 8) over the North Branch that was almost identical with the Archer Avenue bridge,

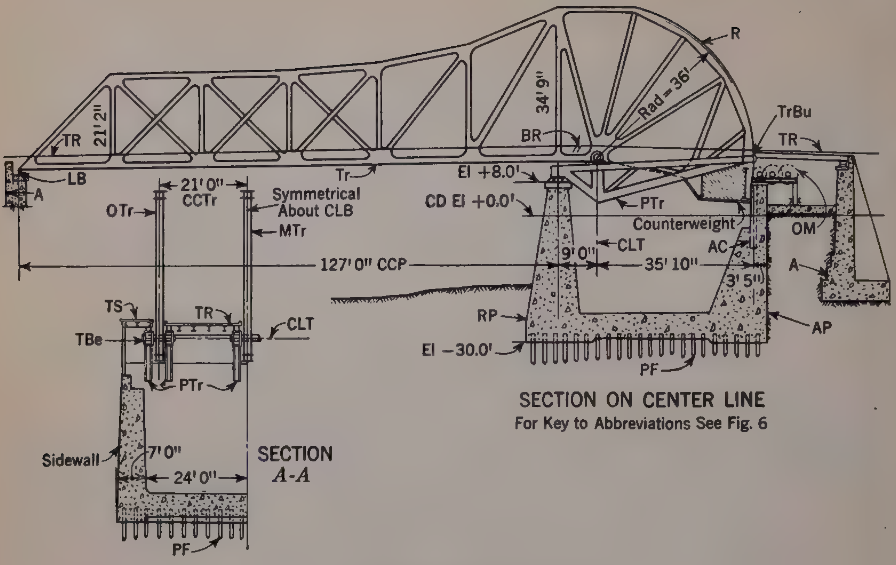


FIG. 8.—WEST KINZIE STREET (1909)

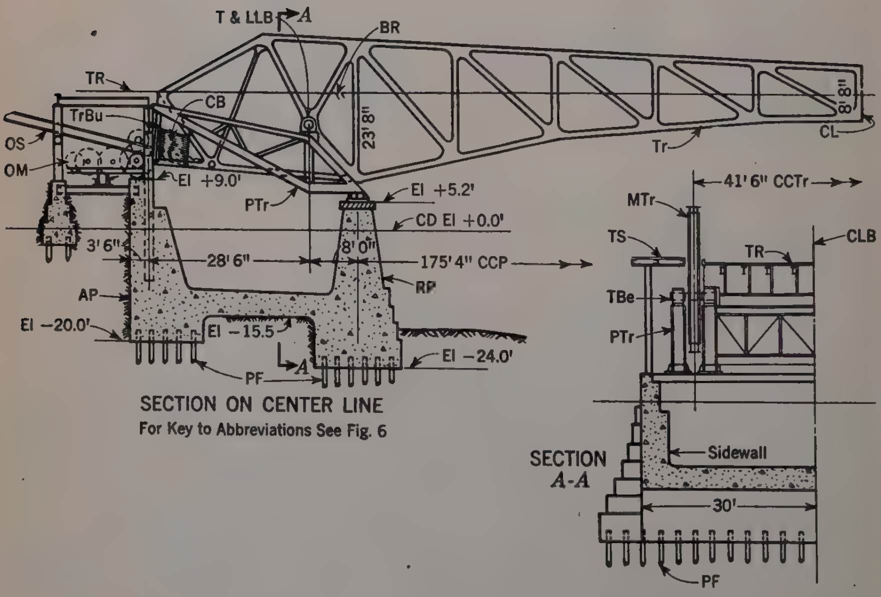


FIG. 9.—WEST ERIE STREET (1910)

previously described, with some improvements in the barrier. Two more double-leaf bridges were built, one at North Avenue over the North Branch and the other at North Halsted Street over the North Branch Canal. These showed only minor variations from previous practice.

In 1910 a double-leaf bridge was built at West Erie Street (Fig. 9) over the North Branch that was radically different from previous practice. The grade of the roadway is about 6 ft higher than on the previous bridges; hence it was feasible to approximate the outline of an arch. Two trusses were used instead of three, as previously, without overhead bracing. They extend only about 10 ft above the roadway at the rear ends. Due to reduction in depth of the tail end, the rear external pin rack was no longer usable, so the bridge was operated with operating struts. These struts were connected to pins cantilevered out sidewise from the bottom chords just ahead of the counterweight box. The machinery gear trains were placed in the rear of the bridge, as before, but at the sides just back of the trusses with the main pinions engaging racks on the struts held in contact therewith by guide rollers. Each train is assembled complete, including motors, in one unit and mounted upon a unit frame to keep parts in permanent alinement. Similar to earlier bridges, the trunnion supports consist of trusses, with the difference that the truss shape now is quadrilateral rather than an inverted triangle. This change was made to accommodate the heavier loads, this bridge having only four supports per leaf rather than six as with the three truss bridges.

The Improvement Period

Between 1908 and 1911 several bridges were designed but it was not until a bond issue was passed in the fall of 1911 that funds were made available for their construction. This gave considerable time to develop new ideas and as a result there was a distinct change with the introduction of this new group of bridges. During this period the city became increasingly conscious of the appearance of its structures and, as a result, the bridges have been given considerable study to enhance their appearance. In 1909 plans were drawn for a new bridge at West Lake Street to provide for the Lake Street Elevated Railway in addition to street traffic. A vertical lift bridge was designed but public opinion was so against its appearance that the plan was abandoned. Later in 1913 a double-deck bascule bridge was developed which will be discussed subsequently.

With this group the method of operating the bridge was changed. In 1908 Alexander von Babo, then Engineer of Bridge Design, invented the internal rack for which he filed an application for a patent on June 26, 1908, and which was issued on August 29, 1911. This patent provided for a rack in the plane of the trusses on the trunnion side of the counterweight box with the truss members arranged so that the bridge could rotate the approximately 80° required without any of the truss members fouling the pinion shaft of the gear train which drives the bridge. The radius of this rack is necessarily considerably less than that of the external pin rack used in the older bridges; but the science of machinery design and manufacture had advanced sufficiently to take care of the larger rack stresses involved.

In this same patent another feature was incorporated, called the cross girder, which consisted of a member being inserted through the truss to carry the inside trunnion journals. This girder was supported in turn upon girders parallel to the trusses, spanning from the front to back piers. In later years, this feature led to a suit for infringement by a competitor who claimed this same feature was incorporated in a patent granted to him earlier than Mr. von Babo's patent. This claim of both patentees was doubtless void as a feature similar to it was incorporated in a bascule bridge over the Tiber River built near Rome, Italy.[3] This cross-girder feature is only practical with two-truss bridges, as it would be impracticable to carry the heavy reactions of a truss on a girder except near the ends. The fixed part-floor system consists of stringers, supported upon this girder, passing over the counterweight box to supports over the anchor pier, instead of being framed between floor beams carried by the longitudinal supports for the inner trunnions formerly used.

Another change was made in the counterweight by forming the box large enough so that the necessary weight could be obtained with stone concrete, or, with only slight amounts of heavier aggregates to produce the weight required to adjust the center of gravity to its proper location to balance the remainder of the bridge.

The truss outlines were made more graceful by the use of trusses of the pony type. They were only deep enough at the rear ends to provide an economical cross section for the truss members, and then they dipped gracefully toward the center of the channel. The bottom chords were also given as much arching as the required underclearances prescribed by the U. S. War Department would permit. At this time the government specified a 16.5-ft clearance above Chicago Datum (normal water level) for 85% of the clear channel from which point the lower chords could be sloped downward.

A machinery girder was placed outside of, and parallel to, the longitudinal girder supporting the cross girder, at a distance of 6.5 ft, which girder, together with the longitudinal girder, supported the machinery gear train. The gear train, in turn, was mounted on a cast-steel pedestal base. These girders also transmit the couple of the wind and operating moments to the masonry at the front and rear piers. Fig. 10 shows the East 92d Street bridge.

With this group of bridges another new feature appeared in the substructure. At West Washington Street, completed in 1913, it had not been feasible to use piles to support the tail-pit boxes, due to the presence of the street railway tunnel under the river. Rectangular piers were sunk from the bottom of the piers to a point well below the tunnel where they rested on a hard clay stratum at El. −57.0 (Chicago Datum), capable of carrying a greater unit load than at higher levels. At the West Chicago Avenue bridge (completed in 1914, see Fig. 11), cylindrical piers were sunk to El. −82.0, where they were founded on hard clay. The spacing was sufficient to permit a future double-track subway tube to pass between them. Reinforcing trusses then had to be placed in the front and rear walls of the tail pit to carry the weight.

Following the bond issue of 1911, six bridges were placed under contract in rapid succession—five of the new double-leaf bascule type and one, a single-

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[3] *Industries*, London, England, Vol. 14, 1893, p. 316.

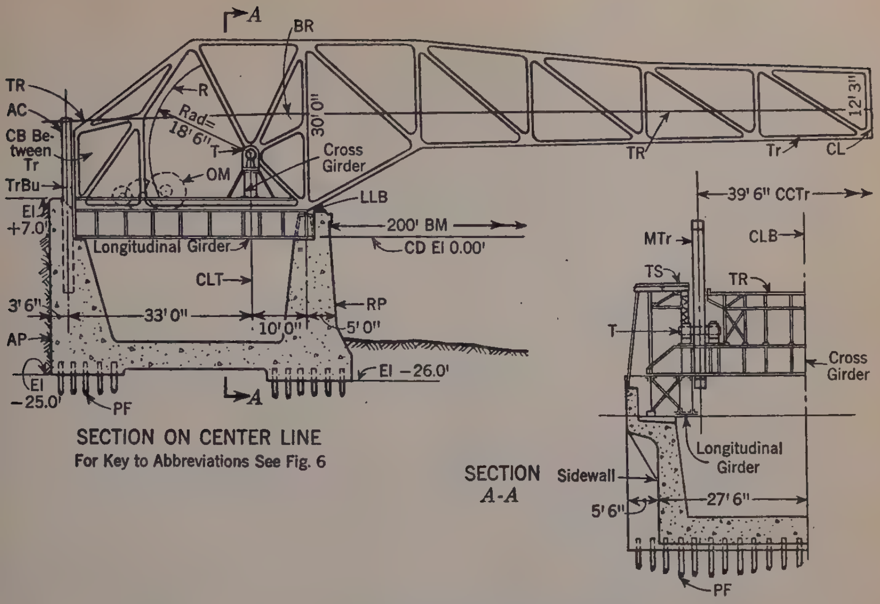


Fig. 10.—East 92d Street Bridge (1913)

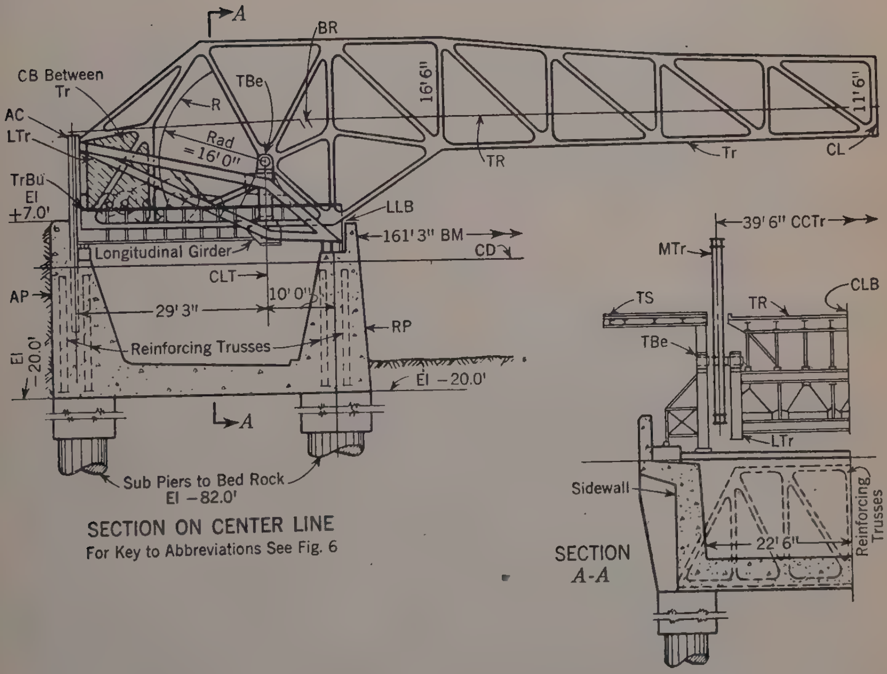


Fig. 11.—West Chicago Avenue Bridge (1914)

leaf bascule at West 35th Street, which was a duplicate of the Archer Avenue and Kinzie Street bridges previously described.

Of the five bascules, two—the Washington Street and Chicago Avenue bridges—had enclosures of improved appearance. In the case of the Washington Street bridge, the house was covered with sheet copper, and the Chicago Avenue bridge houses and enclosures were made of concrete, suitably ornamented. In the case of the bridges following these, extensive architectural treatment was applied to the houses and enclosures, first in concrete and later in stone or other durable materials.

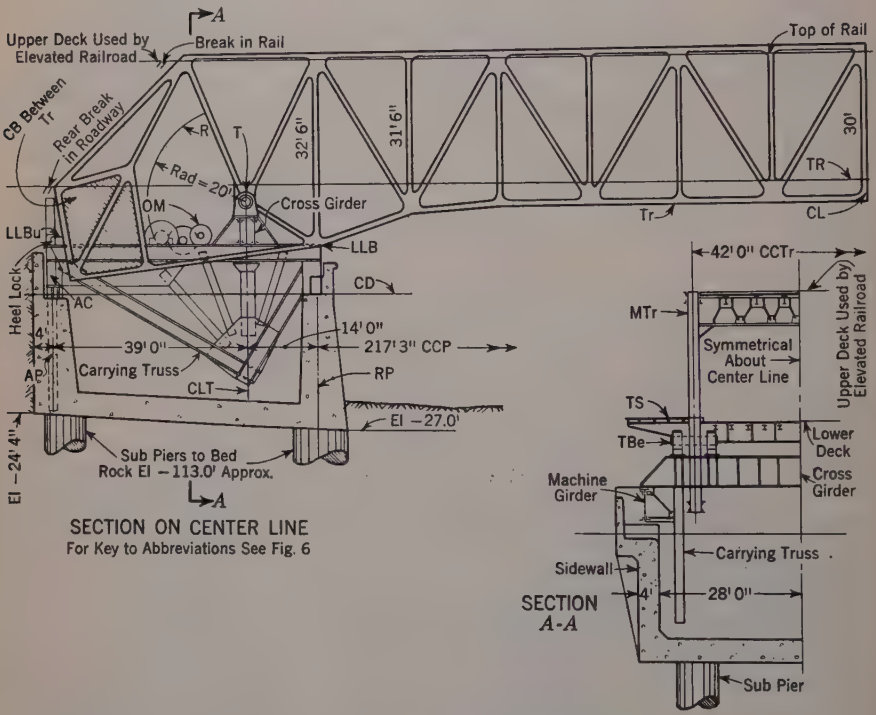


Fig. 12.—West Lake Street Bridge (1914)

In 1913 plans were prepared for a double-deck, double-leaf bascule bridge at West Lake Street (Fig. 12) to carry the elevated railway in addition to the street. Except for the necessary through-truss outline, this bridge had most of the features of the five previous single-deck bridges, such as the cross girder, internal rack, a longitudinal support for the cross girder (a triangular truss instead of a girder), machinery girder, unit machinery frame, and counterweight box with relatively light materials. Lead slag was used instead of stone to produce a concrete weighing 165 lb per cu ft. About the only unusual feature in this case was that everything was heavier, as witnessed by the fact that the trunnion load was about double that at the East 92d Street bridge, the heaviest to that time. With these very heavy loads it was decided to rest the tail pits on cylinder piers carried to solid rock at El. −113.0 and with a clearance of 36 ft to provide for a double-track subway at some future date. Chicago's subway has since utilized this space for a single bore of the Dearborn Street subway system.

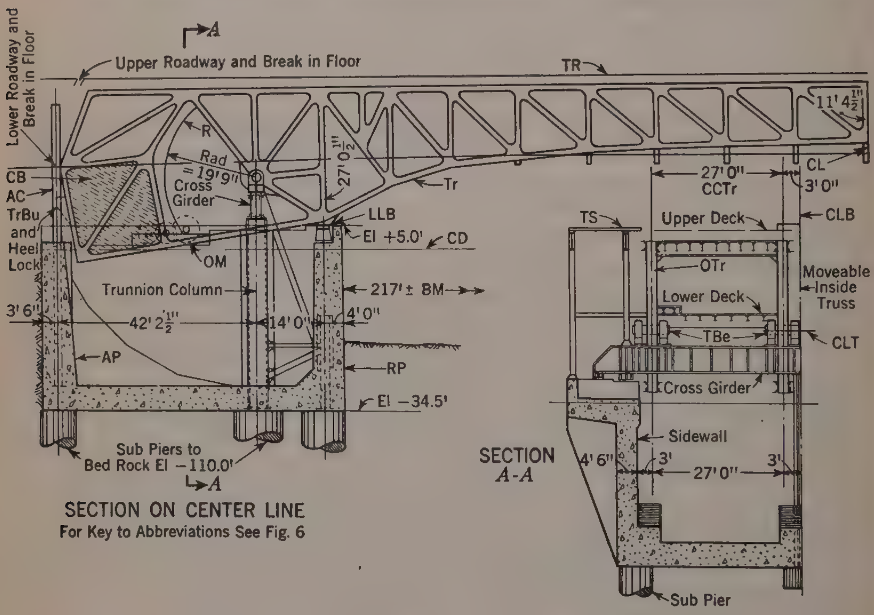
With single-deck bridges it had always been practical to have the break in floor of the deck on the river side of the trunnion so that the raised leaf provided a barrier in the street. With a double-deck bridge, this was not possible so that a rear break in the floor was necessary, which does not provide a natural barrier in the street. Accordingly, it was necessary to provide some kind of a barrier at each end of the bridge. These were not put in at once but were installed at a later date. First, there was a semi-rigid type consisting of a timber pole that could be lowered into place and which would slide up an inclined grade if struck. Later a yielding barrier of the net type was developed, which plays out against friction drums when struck.

In many of the earlier bridges street traffic was accommodated during construction with a temporary bridge, usually of the so-called pontoon type. A truss span was supported at each end upon pile supports, and a scow placed beneath and near one end. By pumping water out of the hull, the truss at this end could be raised and the span could be rotated about a pin at the other end to clear the channel for navigation. At Lake Street, however, the presence of the elevated railway made this impracticable. After attempting to detour via other streets it was decided to maintain traffic by leaving the old swing bridge in place, and to support the approaches upon shoring. The length of the movable span of the new bridge was designed so that it could be erected in the vertical position clear of the ends of the swing bridge, and, as the bridge was erected, sufficient floor system was omitted to allow the elevated trains to pass through. When the leaf was ready to lower, traffic on the elevated railway was interrupted, the old swing bridge was turned on to the center pier, and the center section was cut out. The new bridge leaves were lowered after the omitted section of floor had been placed, the decking for the elevated railway was placed, and traffic was then allowed to pass over the new bridge with only a few days of interruption. No attempt was made to maintain traffic on the lower or street level.

Due to the fact that the bridge was balanced about the trunnion for dead load and had a rear break in the roadway, it was possible for live load to pass on to the rear ends only, which might place live load on the machinery brakes. Accordingly, heel locks were installed to take whatever live-load reaction might come on to them. These consist of toggle levers between the rear pier of the tail pit and a shoe on the rear of the counterweight box that were forced in place by machinery.

In 1922 a similar bridge was completed at North Wells Street to accommodate the North-Western Elevated Railway. In this bridge the cross girder was supported directly upon a column embedded in the side-wall of the tail pit, with a sub-pier under the tail pit carrying the load down to rock at El. −117.0. This obviated the necessity of providing a heavy structural truss to distribute the heavy load to front and rear walls, the loading being 25% greater than at the Lake Street bridge.

Short girders framed into these trunnion columns spanning to the rear wall to carry the machinery gear train. These were not so heavy as formerly due to the omission of the bridge weight and the shortened span. With this bridge the machinery stresses induced by a pressure of 20 lb per sq ft on the upraised leaf were considered rather beyond practical bounds, so it was decided to scale it down to 16 lb per sq in. for the gear train, but to attach a brake directly to the tail end of the bridge trusses. This brake was to be of the pneumatic type and to consist of shoes working on a curved rail set in the masonry. After further consideration it was thought not to be very practicable, because of variable operating characteristics from moisture conditions; hence the regular machinery was left to handle extreme conditions. There has been some distress in the gear train, which has been corrected by replacing parts with higher strength steels than those originally used. Traffic maintenance during the erection of the trusses was similar to that at Lake Street, but with the added feature that the lower-deck traffic was maintained as well as the elevated traffic. The change over from the old swing bridge to the new bridge was made in 72 hr for elevated traffic, but the lower-level traffic was interrupted for about a month.



SECTION ON CENTER LINE
For Key to Abbreviations See Fig. 6

SECTION *A-A*

FIG. 13.—NORTH MICHIGAN AVENUE BRIDGE (1920)

In 1920 the Michigan Avenue bridge was completed (see Fig. 13). This was the most unusual bridge built up to that time. As Michigan Avenue bridge was to be the link between the boulevard system of the north and south sides of the city, it was considered that a four-lane structure would not be sufficient. Furthermore, to separate commercial traffic from boulevard traffic, it was necessary to have two levels—the upper for boulevard traffic and the lower for commercial traffic. Accordingly, it was deemed necessary to have an upper level with six lanes and a lower level with four lanes. It was decided to use four trusses spaced 27 ft, 6 ft, and 27 ft, with separated roadways on both levels for each direction of traffic. In effect, there were two bridges, side by side, connected with diaphragms, that could be cut out to permit one half to be operated while the other was put up in the air for repairs. (This contingency has not been used in 20 years, mainly because necessary repairs have been on the deck which had to be left down to work upon.) The two bridges were supported upon separate cross girders supported near the center line of the bridge on columns to the floor of the tail pits and on trunnion columns in the side-walls. Yielding barriers were provided to protect traffic at both ends on both levels.

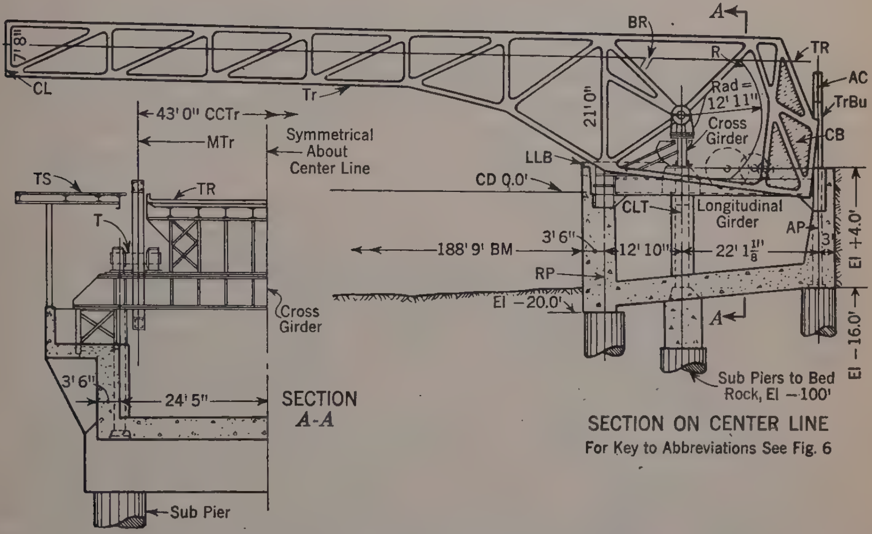


Fig. 14.—West Madison Street Bridge (1922)

Machinery for this bridge consisted of two gear trains per leaf, one at each outside truss. Due to the importance of this bridge it was considered desirable to have an extra pair of motors for stand-by use if the two regular motors should fail; all four motors were grouped in a unit at the rear of the bridge with a clutch to select the pair to be used at any time.

In 1922 another novel type of bridge was completed at West Madison Street (see Fig. 14). At this location there was a clear span of 188.75 ft and, although the street grade on the west side of the river was reasonably high, it was not high enough to permit a deck type of bridge and yet leave a sufficient depth for the trusses. Accordingly, a new idea was developed, called the "railing-height truss," which means that the top of the top chord is at about the same elevation as the top of the sidewalk handrail, about 4 ft above the walk. This gives the appearance of a deck structure from adjacent streets, as the top chord cannot be seen and, by the use of wide sidewalks, the trusses do not obstruct the walkways. Incidental benefits are that vehicles out of control on the roadway cannot get on to the sidewalk to crash into the river or injure pedestrians, and pedestrians are effectively barred from "jaywalking." By the use of nickel steel for main truss members, the sections were kept within reasonable bounds. Supports for the bridge were furnished by cross girders and trunnion columns as previously described for Wells Street.

Between 1911 and the end of 1923, thirteen city-type bascule bridges were built, three double-deck double-leaf bridges, nine single-deck double-leaf bridges, and one single-deck single-leaf bridge. The Sanitary District also built one single-deck double-leaf bridge on the South Branch at West Jackson Street. The total, to this time, was twenty-four city-type bridges.

The Refinement Period 1923–1941

In 1920 funds were becoming scarce, with the result that in 1923, 1924, and 1925 no more bridges were added to the list. However, between 1924 and 1930 several more bond issues were approved by the electorate and in 1926 bridge development was resumed. This period, however, did not produce the radical improvements that marked the years 1911 to 1922. In 1924, when the 100th Street bridge over the Calumet River (see Fig. 15) was proposed, it was decided to avoid all patents, even the internal rack patent held by Mr. von Babo. Mr. von Babo's patent had been used by the city without compensation because he developed it while he was a city employee, although the patent had never been assigned to the city. To overcome this complication, a rack was attached to the side of the truss instead of in the plane of the truss. Instead of the cross

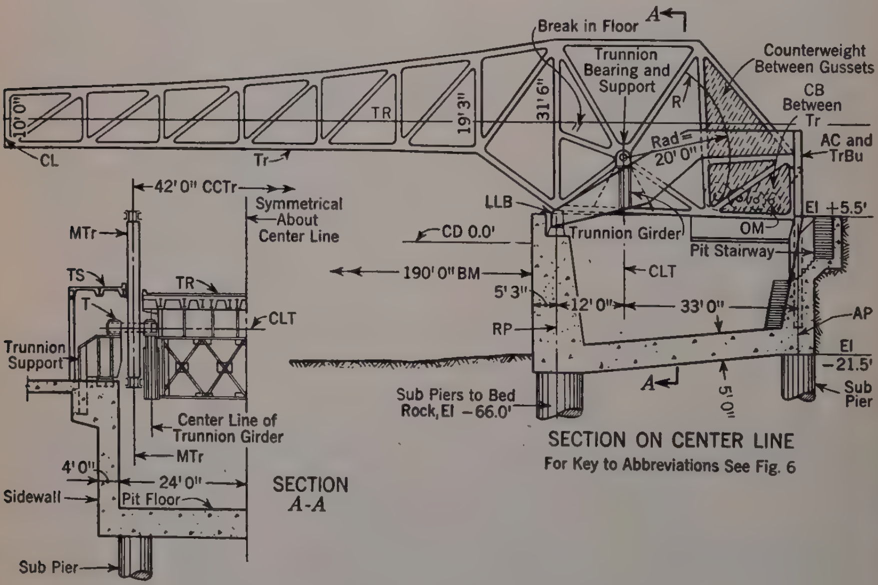
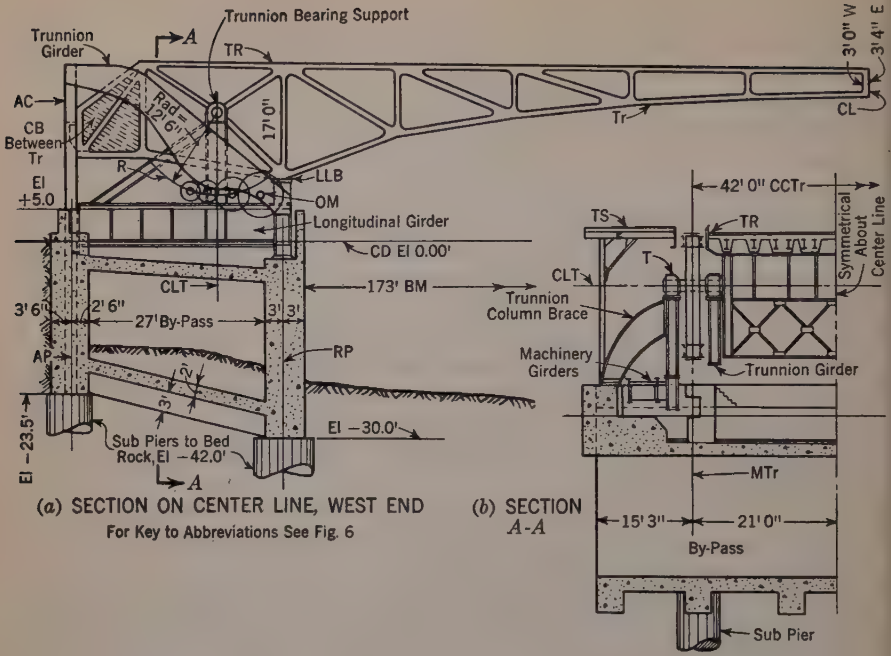


Fig. 15.—East 100th Street Bridge (1924)

girder the inner trunnions were supported on a longitudinal S-shaped box girder in which the rear end of the girder passed over the counterweight box, but yet under the roadway to a support over the anchor pier. In principle this was similar to the idea used as far back as the North Western Avenue bridge completed in 1904, where a tapered box girder was used. In this new development, by using curves it was possible to deepen the section for moment under the trunnion. Near the ends where the depth is less than under the load and where only shear governed, the web plates were reinforced with additional side plates. This support was only used for the inside journals, the outer ones being supported upon short stubby cantilever girders anchored in the side-walls and set transversely across the bridge, thus simulating a cross girder with the center section passing through the trusses cut away. A change was also made in the machinery supports, the unit bed frame being now set directly upon the masonry of the side-walls, which are suitably reinforced with metal grillages embedded in the masonry and with holding-down bolts effectively anchoring the machinery into the masonry. Here another of the recommendations in the 1900 Report of the Board of Consulting Engineers was finally, and most effectively, adopted.

One of the bridges of this period, that at West Adams Street, is interesting in that local conditions on each side of the river forced a different tail-end design on the two sides of the river (see Fig. 16). This bridge is set at such a high grade that a deck bridge was feasible; but, due to the shallowness of the rear arms, an internal or side rack was not feasible, so the rack was placed outside and below the bottom chord in the plane of the member. The machinery on the east side rested directly on the masonry as described in the previous paragraph, but on the west side it was impossible to use the usual pit as a by-pass through the masonry because added flow area in the river was required. Accordingly, the outer trunnion supports and the machinery rest on longitudinal girders spanning from front to rear piers, whereas the inner trunnions rest on S-girders. On the east side the outer trunnions rest on trunnion columns and the inner ones on S-girders. The counterweight arm on the west side was so short that only a shallow pit was required, because (1) the by-pass, and (2) a railroad track on the west dock, did not permit extension that way. This made it necessary to reduce the volume of the counterweight box below the most desirable dimensions. To overcome this condition and to provide the weight required, concrete made with steel punchings to produce a unit weight of 275 lb per cu ft, combined with shaped iron castings, and a small amount of lead pigs were used, thus adjusting the center of gravity to produce an over-all unit weight of about 330 lb per cu ft.

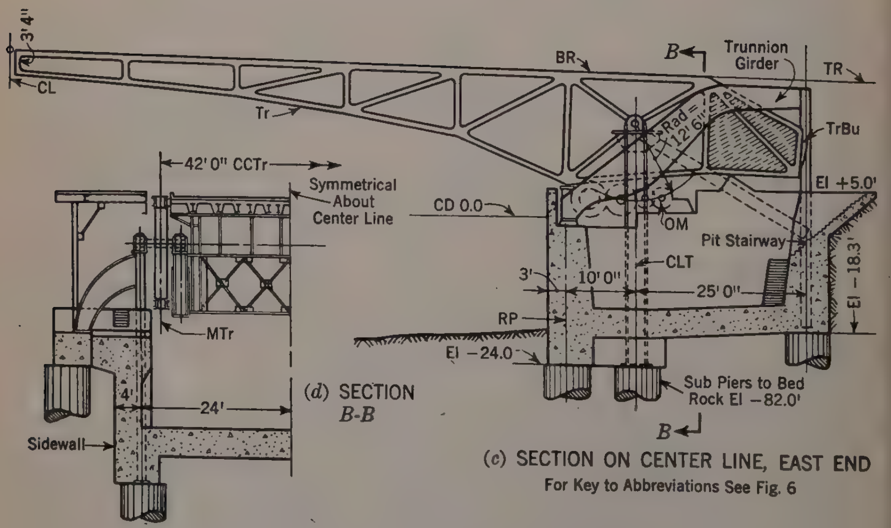
When the South Halsted Street bridge (see Fig. 17) was being designed, a problem was presented because the U. S. War Department urged higher river clearances. Combined with the fact that the street passes through a subway under a railroad just south of the bridge, this requirement would have made almost prohibitive grades on the bridge approach unless the bridge grade were kept as low as possible. The grade was reduced by using three trusses and a very shallow floor system, the floor beams consisting of 30-in. Carnegie beam sections with the total depth of the deck only 3.5 ft from the roadway grade to



(*a*) SECTION ON CENTER LINE, WEST END

For Key to Abbreviations See Fig. 6

(*b*) SECTION *A-A*



(*d*) SECTION *B-B*

(*c*) SECTION ON CENTER LINE, EAST END

For Key to Abbreviations See Fig. 6

FIG. 16.—WEST ADAMS STREET BRIDGE (1926)

the underclearance. As this bridge has two three-lane roadways, it does not give the same constricted appearance as the older three-truss bridges, with only 18-ft roadways.

These same plans were used at South Ashland Avenue bridge, a mile west of the South Halsted Street bridge (1938), not on account of local conditions but in order to use plans that would fit the location and provide a job under PWA without the delay of preparation of new plans. These plans, with the shallow floor, however, had the advantages of keeping the hump in the street profile at a minimum.

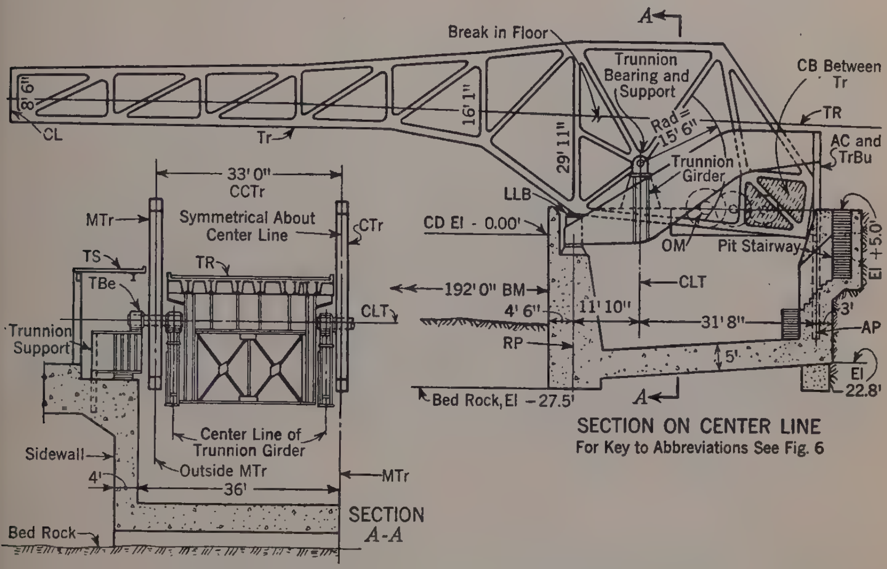


FIG. 17.—SOUTH HALSTED STREET BRIDGE (1933)

The railing-height truss idea, first used for the Madison Street bridge (1922), was used again at the North Wabash Avenue bridge (1930) and the North State Street bridge (1942). However, silicon steel was used instead of nickel steel. The Wabash Avenue bridge (Fig. 18) was awarded the first prize by the American Institute of Steel Construction as the best looking bridge completed in 1930 in the class costing more than $1,000,000. This bridge has a six-lane divided roadway. The center island was used as a protection for a row of T-shaped trolley supports, thus avoiding the necessity for arches over the roadway. These T-shaped trolley arches were preferred to rather flat arches otherwise required.

The North State Street bridge (Fig. 19) is a three-truss bridge because it provides for eight lanes of traffic, and it is not feasible to design floor beams to span the width required for two additional lanes, without a further rise in grade. With such wide roadways the old danger that existed with four-lane roadways of vehicles running head on into the center trusses is not so great; at least, it is not as bad as crashing head on with traffic traveling in the opposite direction at speeds varying from 30 to 40 miles per hr. This bridge has the further innovation of roller bearings instead of phosphor-bronze bushed bearings. These bearings also have spherical ball seats so that they adjust themselves for correct alinement in case of unequal deflection of the supporting structures. This feature reduces bearing friction to a minimum.[4]

Some other new ideas are incorporated in the design of the South Canal Street bridge. The design calls for a double-leaf bridge with a four-lane roadway of the open grating type. There is a great economy in this type of floor for bascule bridges, because the saving in weight of the floor not only reduces stresses in the truss members, but effects a reduction in weight of the counterweight of two to three pounds for each pound saved in the floor weight. Frequently, due to limited space for the tail end, this slight increase in weight can only be attained by the use of heavy unit-weight material which is usually more expensive than plain concrete that could be used otherwise; such a case was described previously in relation to the Adams Street bridge. Another innovation in this bridge is the use of hydraulic motors, instead of the usual electric motors, for driving the gear train. The prime power will be electricity as in the past but the electric motors will operate at constant speed driving an oil pump. By varying the displacement of oil by this pump (which displaced oil is used to drive oil motors at the gear trains), the speed of the bridge can be controlled more effectively than with electric motors which require considerable control equipment to handle the large currents involved. In this case, a simple push-button control is all that is needed to start and stop the constant-speed electric motors.

A total of 39 city-type bridges have been constructed in Chicago. Some of these were built by governmental bodies other than the city. The two on the Outer Drive at the mouth of the Chicago River were built in 1937 by the Chicago Park District and were designed by a local firm of consulting engineers, but following, carefully, the city's practice on fundamental details. These bridges provide two four-lane roadways on the upper level, and the lower level provides for railroad tracks if future developments in this part of the city require them.[5]

As a side issue, not exactly within the province of this paper, is a note concerning the construction of the only movable bridge, not of the bascule type, constructed in Chicago in the past 25 years, at South Torrence Avenue over the Calumet River. A vertical lift type was used, at a location where the center line of the river channel forms an angle of 50° with the center line of the street. Here the U. S. War Department required a clear channel of 200 ft with a 21-ft headroom for the full channel. Hence a bascule bridge would have had to have a clear distance between masonry of about 320 ft. As this bridge is in a commercial or manufacturing district, a skewed vertical lift bridge was not considered out of place and comparative estimates indicated the latter could be built for less than half the cost of a bascule.[6]

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[4] *Civil Engineering*, March, 1942, p. 140.

[5] *Engineering News-Record*, April 15, 1937, p. 546; also April 22, 1937, p. 583.

[6] *Ibid.*, November 17, 1938, p. 618.

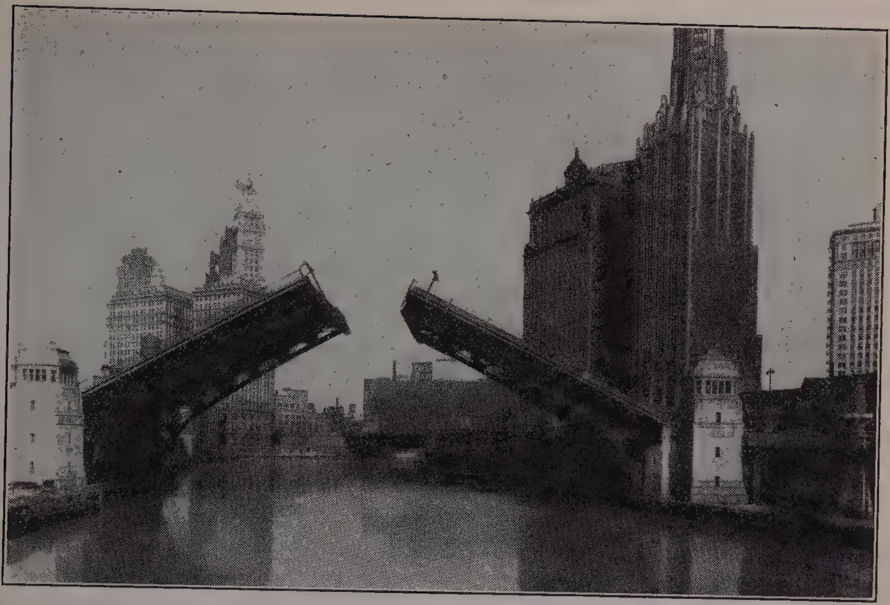

Fig. 18.—North Wabash Avenue Bridge (1930)

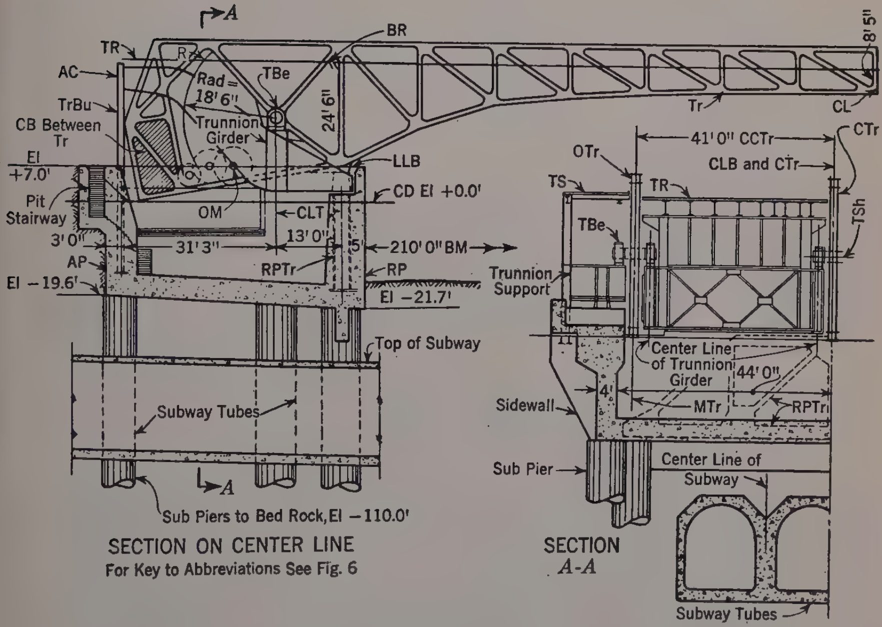

Fig 19.—North State Street Bridge

CONCLUSION

Figs. 5 to 19 indicate the development of the Chicago type bascule bridge from the Clybourn Place and Division Street bridges, down through the years to the latest structures. Table 1 gives statistical data, which, by comparison, show the development of this type of bridge better than meager words. This type of bridge has proved highly satisfactory and has solved most of the problems existing previous to its development, as discussed in the historical part of this paper previous to 1900.

TABLE 1.—DATA AND CHRONOLOGY, CHICAGO BASCULE BRIDGES
(Dimensions Are in Feet)

Year	Location (see Fig. 1)	Fig. No.	Bridge type[a]	Clear span	Between trunnions[b]	River channel	Bridge width	Trusses	
								No.	Type
1902	W. Cortland St. (Clybourn Place)	5	SD/DL	115.0	128.0	92.0	60.0	3	Through
1903	E. 95th Street	..	SD/DL	140.0	154.0	120.0	60.0	3	Through
1903	W. Division St. (Canal)	6	SD/DL	135.0	149.0	90.0	60.0	3	Through
1904	W. Division St. (River)	7	SD/DL	155.67	172.67	100.0	60.0	3	Through
1904	N. Western Ave.	..	SD/DL	184.58	205.58	121.0	60.0	3	Through
1906	W. Archer Ave.	..	SD/SL	121.0	136.0	100.0	60.0	3	Through
1907	W. North Ave.	..	SD/DL	153.67	172.67	128.0	60.0	3	Through
1908	N. Halsted St. (Canal)	..	SD/DL	184.58	205.58	90.0	60.0	3	Through
1909	W. Kinzie St.	8	SD/SL	121.0	136.0	100.0	60.0	3	Through
1910	W. Erie St.	9	SD/DL	170.33	191.33	140.0	60.0	2	Pony
1913	W. Washington St.	..	SD/DL	170.58	197.33	150.0	57.25	2	Pony
1913	W. Grand Ave. (Indiana St.)	..	SD/DL	162.75	188.75	140.0	60.0	2	Pony
1914	E. 92d St.	10	SD/DL	200.42	228.0	190.0	60.0	2	Pony
1914	W. 35th St.	..	SD/SL	120.0	136.0	120.0	60.0	3	Through
1914	W. Chicago Ave.	11	SD/DL	161.25	188.75	150.0	60.0	2	Pony
1916	W. Lake St.	12	DD/DL	209.25	245.25	190.0	70.0	2	Double deck
1916	W. Webster Ave.	..	SD/DL	161.25	188.75	120.0	60.0	2	Pony
1916	W. Jackson Blvd.	..	SD/DL	173.5	202.27	147.0	64.0	2	Deck
1917	W. Belmont Ave.	..	SD/DL	161.25	188.75	75.0	60.0	2	Pony
1919	W. Monroe St.	..	SD/DL	165.5	192.79	164.0	60.0	2	Pony
1920	N. Franklin-Orleans St.	..	SD/DL	220.0	251.83	178.0	62.0	2	Pony
1920	N. Michigan Ave.	13	DD/DL	220.0	256.0	200.0	91.79	4	Double deck
1922	N. Wells St.	..	DD/DL	231.0	268.0	220.0	72.0	2	Double deck
1922	W. Madison St.	14	SD/DL	188.75	221.23	168.0	72.0	2	Rail height
1926	E. 100th St.	15	SD/DL	200.0	233.5	190.0	62.0	2	Pony
1926	W. Adams St.	16	SD/DL	173.0	199.0	151.0	64.0	2	Deck
1928	E. 106th St.	..	SD/DL	220.0	250.6	192.5	62.0	2	Pony
1928	N. LaSalle St.	..	SD/DL	210.0	242.0	200.0	86.0	2	Pony
1929	N. Clark St.	..	SD/DL	215.0	245.5	200.0	72.0	2	Pony
1929	W. Roosevelt Road (Twelfth St.)	..	SD/DL	170.0	204.0	170.0	90.0	2	Deck
1929	N. Damen Ave. (N. Robey St.)	..	SD/SL	120.0	162.23	120.0	62.0	2	Deck
1929	S. Damen Ave. (S. Robey St.)	..	SD/DL	212.0	243.83	140.0	66.0	2	Pony
1930	N. Wabash Ave.	18	SD/DL	232.0	269.0	193.0	90.0	2	Rail height
1932	W. Ogden Ave. (Canal)	..	SD/DL	158.0	191.0	132.0	90.0	2	Pony
1932	W. Ogden Ave. (River)	..	SD/DL	150.0	182.0	130.0	90.0	2	Pony
1933	S. Halsted St.	17	SD/DL	192.0	224.67	165.0	90.0	3	Pony
1937	Outer Drive (River)	..	DD/DL	220.0	264.0	220.0	108.0	4	Double deck
1937	Outer Drive (Canal)	..	DD/SL	70.0	96.5	70.0	108.0	4	Double deck
1938	S. Ashland Ave.	..	SD/DL	192.0	224.67	189.83	90.0	3	Pony
1942[c]	N. State St.	19	SD/DL	210.0	245.0	200.0	108.0	3	Rail height
1943[c]	S. Canal St.	..	SD/DL	224.0	254.5	171.0	68.0	2	Pony

[a] Key: SD = single deck; DD = double deck; DL = double leaf; and SL = single leaf. [b] Distance, center to center of trunnions. [c] Completion deferred until after the War.

The navigation requirements can be met ideally by the proper selection of size and location of the channel, almost without limit. The requirements of street traffic are satisfied by the provision of a structure that can be operated reliably in a minimum of time and which provides a measure of protection to the traffic when open, not provided by a swing or lift bridge. Maintenance is kept at a low figure by avoiding pin-connected linkages except for the trunnions upon which the movable part rotates.

The extra cost (if any) of this type of bridge over swing or lift types has been justified by its far better appearance especially when placed in a congested section of the city. However, the development of this type of bridge has not yet reached a point of stagnation as demonstrated by the various bridges built during the so-called Refinement Period, beginning in 1923, and many more refinements can be expected in the years to come.

In the development of this bridge, exclusive credit cannot and should not be given to any one engineer as it represents a development over a period of more than 40 years during which time many engineers have had an opportunity to add their bit to what has gone before. The complexity of such a structure, involving complicated foundation design conditions, unusual structural conditions, special mechanical and electrical features and, finally, architectural details in operators' houses and enclosures, is beyond the capabilities of almost any one man. Accordingly, credit is given to each and every one ever connected with its development in proportion to his contribution.

However, most of the early development was due to the efforts of the late John Ericson, city engineer, Thomas G. Pihlfeldt, engineer of bridges for more than 40 years, until the time of his death in 1941, and Alexander von Babo, engineer of bridge design for more than 20 years, until his death in 1920.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

EARLY CONTRIBUTIONS TO MISSISSIPPI RIVER HYDROLOGY

Discussion

BY MESSRS. GEORGE O. GUESMER, AND H. T. CORY

GEORGE O. GUESMER,[29] ASSOC. M. AM. SOC. C. E.[29a]—The source and accuracy of original data which are the basis for a study of any subject determine the value of the conclusions reached. Mr. Jarvis' diligence in "digging out" information buried for years in the records of various offices in the Mississippi River Valley has given him an excellent background.

The writer had an opportunity to observe Mr. Jarvis gathering information used in the preparation of this paper. The comprehensiveness of his research was evident not only from the continuous testing and correlation of each new "lead" but also from the new derivations made in the light of basic data assembled in the course of related studies and research projects. Every effort was made to be certain that each item was thoroughly examined and worthy of consideration. Thus, the paper is based on information carefully weighed by the author during selection at the original source.

H. T. CORY,[30] M. AM. SOC. C. E.[30a]—It is most unfortunate that a wholly unjustifiable adoration of the Golden Calf Precision has blinded so surprisingly many to the solid substantiality of what early investigators and constructors did and knew. That Mr. Jarvis should have realized the Humphrey and Abbott early Mississippi records merited thoroughgoing examination by modern critical methods and should have discovered that the endeavor was very much worth while—is reassuring. It is to be hoped that his example will be followed, and followed frequently.

Sometimes the rejection of old-time "crudely observed" data approaches the tragic. An example coming within the writer's personal experience is a case in

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NOTE.—This paper by C. S. Jarvis, M. Am. Soc. C. E., was published in March, 1942, *Proceedings*. Discussion has appeared in *Proceedings*, as follows: September, 1942, by Messrs. Robert Follansbee, and R. W. Davenport; November, 1942, by Charles Senour, M. Am. Soc. C. E.; and December, 1942, by John C. Hoyt, M. Am. Soc. C. E.

[29] Formerly Engr., U. S. Engr. Office, St. Paul, Minn.

[29a] Received by the Secretary January 5, 1943.

[30] Superv. Engr., Reconstruction Finance Corp., Los Angeles, Calif.

[30a] Received by the Secretary January 11, 1943.

point. In 1912 he had occasion to meet an old gentleman then 94 years old, who had a remarkably keen memory, supplemented by scattered notes preserved in old notebooks, which were not diaries in any sense.

Among the many impressive and stirring events discussed, the most striking was his being taken, when a youth, by an uncle on a trip to Cincinnati, Ohio. At this time there must have been a notable drought for he vividly recalled having seen the steamboats resting crosswise on the Ohio River bed with the only water in the river running under and not touching the boat bottom.

The writer naturally grilled the old gentleman thoroughly, and was convinced he had described exactly what he had seen at Cincinnati on that trip. Absurd as it may seem to those who have not, more than once, been dumbfounded by the "incredibility" of high and of low extremes of stream discharge and yet lived to see such extremes reached if not exceeded, the writer is convinced that about 1835 the discharge of the Ohio River at Cincinnati fell to, relatively speaking, zero. Several times the incident has been brought to the attention of the federal water resources officials during the thirty years since 1913 but apparently to no purpose. Such "stream gaging" is certainly very crude, and yet —.

After all, there is no doubt that the best way to bring proper attention to such cases of "rejected data" is to follow Mr. Jarvis' example and to present carefully prepared papers to the Society for its information and discussion. The present paper may well be taken as a model.

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DISCUSSIONS

PROFILE CURVES FOR OPEN-CHANNEL FLOW

Discussion

BY DWIGHT F. GUNDER, ESQ.

DWIGHT F. GUNDER,[19] ESQ.[19a]—When preparing this paper, the writer realized that he was venturing into a field where he was almost entirely inexperienced and, consequently, where his approach was somewhat uncertain. His one hope was that some discussion might result which would clarify the rather confused status of the type 3 curves appearing in open-channel flow. The excellent contributions of the discussers have certainly justified this hope.

Before considering these discussions individually, the author wishes to acknowledge an error in the paper. The point of inflection for the Bazin and Ganguillet-Kutter relations by the straightforward but extremely tedious method of determining the point of inflection was located by finding $\frac{d^2y}{dx^2}$ and solving this equation by approximation. In so doing, a serious error of computation was committed. (Few mathematicians, if such the writer could be called, can do arithmetic.) The much more elegant treatment of this part of the problem by Messrs. Bakhmeteff and Feodoroff clearly showed a discrepancy. In light of this and the experimental findings of the same two discussers, the second of the conclusions of the "Synopsis" should be written as follows:

(2) Use of the differential equation of gradually varied flow at depths below both the normal and critical depths should be made with reservations until more experimental work has been done in this region of high velocity flow. The author deeply regrets the inclusion of this error but is glad that it was caught in the discussions.

The suggestions of Messrs. Rohwer and Rouse that certain other specific cases be considered are excellent although they provide rather severe difficulties unless one is to adopt approximate methods which, in turn, are essentially

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NOTE.—This paper by Dwight F. Gunder, Esq., was published in April, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1942, by Carl Rohwer, Assoc. M. Am. Soc. C. E.; September, 1942, by C. J. Posey, Assoc. M. Am. Soc. C. E.; October, 1942, by Messrs. J. C. Stevens, and Boris A. Bakhmeteff and Nicholas V. Feodoroff; and November, 1942, by Hunter Rouse, M. Am. Soc. C. E.

[19] Research Engr. and Associate Prof., Math. and Civ. Eng., Colorado State College, Fort Collins, Colo.

[19a] Received by the Secretary January 15, 1943.

those now used. Further, it seems from the evidence presented by Messrs. Bakhmeteff and Feodoroff that further refinement in the analysis should await additional experimental data relative to the various roughness factors and to the profile curves themselves. As soon as some one obtains some rather complete and reliable data on this particular region of flow, it may be advisable to attack the problem using, say, the Kármán-Prandtl expression for the Cheźy C, as is suggested by Professor Rouse. The mechanical difficulties attendant upon attaining a closed solution to this problem are great but not necessarily prohibitive.

With regard to the discussion of Mr. Stevens, only one point probably should be mentioned. In the solution of the flow equation, as Professor Rouse has stated so well, the final form was presented only for what it might be worth to those who had use for it. As a result, this equation was stated in terms of the original variables and parameters with no attempt at simplification, leaving such simplifications or alternative forms to those interested in using the result. The feature which the writer thought obvious, but which apparently Mr. Stevens (and hence perhaps others) missed, is that the differential Eq. 15 can be put in the dimensionless form

$$d\xi = \frac{\eta^{1/3}\,(\eta^3 - \eta^3_c)}{\eta^{10/3} - 1}\,d\eta \ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

in which $\eta = \dfrac{y}{y_o}$, $\eta_c = \dfrac{y_c}{y_o}$, $\xi = \dfrac{S_o\,x}{y_o}$. Eq. 17 then reduces to

$$\xi = \eta + 0.306\,\eta^3_c + 0.3\,K_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

in which K_1 is dimensionless and a function of η_c and η only. It is thus seen that the plot of the ξ,η-curves, which give the x,y-curves directly, is a one-parameter family with the parameter η_c. Furthermore, K_1 may be separated into two parts, one of which (say, K_2) is independent of η_c, and the other of which (say, K_3) consists of three or four relatively simple terms involving η and η_c. Thus, the construction of the table which Mr. Stevens recommended as a sentence for the writer could be accomplished in quite a restricted space. It appears as though the "professor" had overestimated the ability of one of his "students." This situation seems inseparable from the learning process.

The writer wishes to acknowledge especially the fine work of Messrs. Bakhmeteff and Feodoroff in correlating the discussion of the various resistance factors and correcting the writer's error, as well as in giving some intimation by experimental studies of the true state of affairs as regards the type 3 curves in open-channel flow.

Finally, the author feels that the excellent discussions have contributed more to the problem than did the original paper, the only merit of which is perhaps that it brought forth these discussions.

Correction for *Transactions:* In November, 1942, *Proceedings*, page 1647, Fig. 8, change "R" to "R" in three places; and, in Fig. 8(*b*), change "$\dfrac{\mathsf{R}^{1/2}}{n}$" to "$\dfrac{R^{1/6}}{n}$". In April, 1942, *Proceedings*, page 539, Fig. 1, y_o and y_c are measured vertically.

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DISCUSSIONS

STATISTICAL ANALYSIS IN HYDROLOGY

Discussion

BY MESSRS. R. W. DAVENPORT, EDWARD J. BEDNARSKI, AND RALPH W. POWELL

R. W. DAVENPORT,[30] M. AM. SOC. C. E.[30a]—An interesting illustration of the significance of plotting flood data, by the method presented by Mr. Beard, is afforded by a study made by the writer. The frequency relation, or curve, of maximum annual floods is developed graphically on probability paper for a certain river measurement station on the basis of available records and an estimated extrapolation of the relation to cover very rare floods. The derived relation is assumed to represent flood occurrences of the future or those that are "in store." For purposes of the study the writer developed the highest 1% of these flood occurrences into terms of 100 rates of discharge selected on the scale of frequency so that each term represented a range of equal frequency of occurrence.

The practical significance of the rates of discharge so selected is that, if a 10,000-yr period is conceived as so ideal that the 10,000 maximum annual floods would plot smoothly on the foregoing curve, the selected rates would represent the 100 highest annual floods in the 10,000 years, or the 100-yr floods, as they are called herein.

Although the average frequency of the selected floods is once in 100 years, it is apparent that if the 10,000-yr period were considered as one hundred distinct 100-yr periods, the distribution of the 100-yr floods among such periods would not be uniform. In fact, some 100-yr periods would have none of the 100-yr floods, some would have one, and some would have two or more. Assuming that rare floods occur fortuitously (it is enlightening to explore the assumption although it may vary more or less from the fact), statistical theory affords a means, through the application of the normal probability (binomial) distribution, to compute the theoretical distribution of the 100-yr floods among the one hundred 100-yr periods. Such distribution would be approximately as follows:

NOTE.—This paper by L. R. Beard, Jun. Am. Soc. C. E., was published in September, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1942, by Messrs. L. Standish Hall, and H. Alden Foster; November, 1942, by Messrs. Paul V. Hodges, and Joe W. Johnson; and December, 1942, by R. H. Blythe, Jr., Esq.

[30] Hydr. Engr., U. S. Geological Survey, Washington, D. C.

[30a] Received by the Secretary November 30, 1942.

100-yr periods	100-yr floods per period	100-yr floods
37	0	0
36	1	36
19	2	38
6	3	18
2	4	8
100		100

The one hundred 100-yr flood values were assigned numbers consecutively from 1 to 100 and, continuing further on the assumption of fortuitous occurrence, random selections were made of numerous (26) sets of 100 numbers within that range. In each set the first thirty-six numbers were taken to represent the maximum floods in the respective thirty-six 100-yr periods in which one 100-yr flood occurred (second term, third column of the foregoing tabulation). The next thirty-eight were taken to represent the nineteen groups of two 100-yr floods which occurred at the rate of two floods per century. The higher of each group of two was selected and the lower discarded.

Following a similar procedure for the groups of three and the groups of four, the list was reduced to sixty-three terms which afford a basis for evaluating the sixty-three greatest of the maximum floods for 100-yr periods in the assumed 10,000 years. This left thirty-seven 100-yr periods of the one hundred in the 10,000-yr period in which the maximum floods were smaller in magnitude than any of the 100-yr floods.

The twenty-six sets of sixty-three flood values obtained as indicated were each arranged in order of magnitude and those of the same numerical order were averaged to produce a master set representing, in a sense quite limited, of course, experience of a 260,000-yr period.

According to the reasoning on which the plotting procedure outlined by Mr. Beard is based, the probability is 0.50 that the median of the 100 greatest floods for 100-yr periods will not be exceeded in any single 100-yr period. The probability that it will not be exceeded in a given year is $(0.50)^{1/100}$ or 0.9931. Thus, in plotting the maximum annual floods for a given 100-yr period, the greatest value would be plotted at 0.9931 rather than at 0.9950 as is contemplated in some other plotting procedures.

The flood data studied showed the following results (in cubic feet per second):

Average median of 100 greatest floods for twenty-six 100-yr periods	656,000
Flood of 0.9931 probability for all maximum annual floods "in store"	655,000
Flood of 0.9950 probability for all maximum annual floods "in store"	680,000

The close agreement of the first two of these results tends to demonstrate the soundness of Mr. Beard's procedure. Similar close agreement between flood values at corresponding plotting points was found throughout the range of frequencies considered. The results illustrate that essentially the magnitude and frequency relation of the array of greatest floods for 100-yr periods as arrived at by random selection can be derived directly from the relation based upon the array of maximum annual floods described in the first paragraph of this discussion.

One would be fortunate to have a record of floods on a stream in the United States for as many as 100 years. The maximum of the record would be a single sample of the many greatest floods for 100-yr periods which in a very long record may, so far as present knowledge goes, cover a rather wide range of magnitude. In the foregoing example the median of these values is 656,000 cu ft per sec. The distinctive feature in the plotting method presented by the author is that the assumed single sample would be plotted in the same position as the long-time median, 656,000 cu ft per sec, rather than at the midpoint of the highest 1% of the maximum annual floods, for which the value in the example is 680,000 cu ft per sec. Thus, in this instance the average discrepancy involved would be about 3.8%.

The distribution of the greatest floods for 100-yr periods would be skew. The plotting position for a single sample may conceivably be influenced by various practical considerations inherent in the particular problem to which the data are applied. For example, the position that would give the smallest average deviation from the ultimate curve, signs considered, would be that corresponding to the mean of the greatest floods for the 100-yr periods. It is evident that this position would be at a higher probability value than that for the median, but it is not subject to ready determination. Moreover, such plotting position, if known, would have the disadvantage that more than one half of a large group of 100-yr maxima would plot below the ultimate curve. The plotting position corresponding to the modal value of the skew distribution would seemingly be similarly indeterminate and would lead to more than half of the 100-yr maxima being plotted above the ultimate curve. Using the median as in Mr. Beard's method, the plotting position is readily determinable, and one half of the maxima would plot below and one half would plot above the ultimate curve.

As suggested in the "Synopsis," the differentiation between plotting a single 100-yr maximum at a percentage of occurrence 0.9950 or 0.9931 is of "small significance compared to inaccuracies existing in the greater part of the hydrologic data available today." It is of even smaller significance when one considers that, in so far as fortuity rules, the sample of the greatest flood experienced in a 100-yr period may vary by a large percentage from the long-time median value of such greatest floods.

The writer believes that the principal value of the method is in its fundamental soundness and its clarifying influence on reasoning and concepts, which influence may spread much beyond the consideration of the matter that may be immediately affected. The author is to be commended for his interesting and valuable contribution.

EDWARD J. BEDNARSKI,[31] ASSOC. M. AM. SOC. C. E.[31a]—The topic of the paper deals mostly with the application of statistical analysis to hydrology in order to obtain from the records of the past an advance judgment on the expectations in the future.

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[31] Structural Engr., Los Angeles, Calif.

[31a] Received by the Secretary December 31, 1942.

At any stage in the development of science there are the events that are predictable and the events that are not. This should not be lost from the sight of hydrologists working as statisticians. The American experience table on mortality gives information on life expectancy for men of every age, but there are no tables giving information as to when any particular individual will die. This problem still belongs to the realm of prophecy and is unpredictable by the theory of probability. The chances that a given person will win the Irish sweepstakes are very small; no one can predict who will win; but one thing is certain—the winner must have a ticket. This fact is predictable. Accordingly as yet no one can predict on the basis of past records the order of magnitude of the flood that might occur in a given locality this or next year.

In connection with this, the concept of "probable maximum flood," as defined by Mr. Ruff[5] and quoted by the author is very important. The flood should be "so large that the chance of its being exceeded is no greater than the hazards normal to all of man's activities." The writer would like very much to know the magnitude of this normal chance.

The literature on the subject shows that the persons confronted with the problem of advanced judgment in matters of hydrology do not consider the situation in this light but try to have some scientific justification for all their decisions. The theory of probability appears to satisfy the need.

There is a tendency in general to accord preference to the solution of the problem if the solution is the result of complicated, lengthy, and tedious computations—carefully, numerically checked. The mathematical theory of probability with regard to statistics offers a fertile field with its three cardinal equations of distribution, supplemented by a host of others supposed to be easier in application and to reflect better the actual trend of the natural phenomena recorded in the past. Not so long ago a general frequency function was offered in the form of an integral expressing a duration curve applicable to cases with one or two boundaries or no boundaries. Incidentally, Professor Slade[3] calls the representation of probability by a duration curve "usual convenience," thus corroborating the author's recommendations for its use.

It was with a sense of deep relief, therefore, that the writer found the simple description of a new way to compute the "plotting positions" (the term used by Mr. Foster[32]). To make the computations more automatic the writer converted the reasoning into a mathematical expression as follows:

$$\sum_{i=1}^{n} (C_{i-1})^m \, (q_n)^{m-(i-1)} \, (p_n)^{(i-1)} = \tfrac{1}{2} \ldots\ldots\ldots\ldots\ldots (6)$$

This expression happens to be a variable component of

$$\sum_{i=1}^{m} (C_i)^m \, q^{m-i} \, p^i = (q + p)^m = 1 \ldots\ldots\ldots\ldots\ldots (7)$$

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[5] *Transactions*, Am. Soc. C. E., Vol. 106 (1941), p. 1453.

[3] "An Asymmetric Probability Function," by J. J. Slade, Jr., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 35.

[32] *Ibid.*, Vol. 99 (1934), p. 1217.

Essentially, Eq. 6 is the same as Eq. 4; but the comparison of Eqs. 6 and 7, in the form presented, shows strikingly the basic difference between the binomial law of distribution and the method of plotting positions, as developed by Mr. Beard. The solution of Eq. 6 for $n = 1$ and for values of m consecutively equal to 1, 10, 25, 50, 100, 200, 300, 500, and 1,000, results in the percentages shown in Table 3 of the paper. Here is a noteworthy circumstance. The approximate solution of these values can be obtained from the equation

$$p = 100 \frac{n - 0.33}{m} \qquad (8)$$

which is almost identical to that presented in 1927[33] by William P. Creager and Joel D. Justin, Members, Am. Soc. C. E., namely,

$$p = \frac{100\ (n - 0.50)}{m} \qquad (9)$$

Eq. 9 in turn is identical to the expression given by Mr. Foster[32] in 1934 as

$$p = \frac{100\ (2\ m - 1)}{2\ n} \qquad (10)$$

in which the notations m and n have the respective meaning of n and m in the foregoing formulas.

Eq. 9 determines,[33] "according to the law of probabilities, the probable percentage of future floods that will equal or exceed a given flood." In the Foster paper, Eq. 10 gives a plotting position[32] "on the %-of-time axis," which is the same. In Table 3 of the paper, the same formula, with a simple substitution of the constant $\frac{1}{2}$ by $\frac{1}{3}$, solves the problem of determining the flood with regard to the expected life of the structure. This is a startling coincidence. Table 6 shows the values of p computed by Eqs. 6, 8 and 9.

TABLE 6.—COMPARISON OF PERCENTAGES (p) COMPUTED BY EQS. 6, 8, AND 9

Eq.	Values of m:							
	10	25	50	100	200	300	500	1,000
6	6.7	2.74	1.39	0.68	0.35	0.24	0.14	0.07
8	6.7	2.68	1.35	0.67	0.35	0.23	0.135	0.067
9	5.00	2.00	1.00	0.50	0.25	0.17	0.10	0.05

Another remarkable property of Eq. 6 as it is represented by Table 1 is that, if plotted on the ordinary cross-section paper, it produces a straight line; but if plotted on the logarithmic probability paper it resembles very much the curve of peaks presented by Professor Slade.[3]

From the reasoning which results in the computation of the data in Table 1, one may conclude that it represents either a duration curve or a curve of medians for maximums of various degrees of magnitudes. It can be represented with a fair degree of accuracy by an equation of a straight line

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[33] "Hydro-Electric Handbook," by W. P. Creager and Joel D. Justin, John Wiley and Sons, Inc., New York, N. Y., 1927, Eq. 70, p. 171.

$$p = a\,n + b \quad \ldots\ldots (11)$$

in which a and b are functions of m. For $m = 10$, $a = 9.62$ and $b = 2.92$.

The values computed according to Eq. 11 are given in Table 7, together with the values of Table 1.

TABLE 7.—COMPARISON OF p COMPUTED BY EQ. 11 WITH TABLE 1

Description	VALUES OF n:									
	1	2	3	4	5	6	7	8	9	10
Table 1........	6.7	16.4	25.8	35.5	45.2	54.8	64.5	74.2	83.6	93.3
Eq. 11.........	6.7	16.32	25.94	35.56	45.18	54.80	64.42	74.04	83.66	93.28

Similarly the "plotting positions" for values of m other than 10 could be computed. In his reference to the formula mentioned by Mr. Foster, Mr. Beard seemingly assumes that the data of Table 1 determine the duration curve. The derivative of Eq. 11 representing it should produce a corresponding distribution curve. The derivative is a constant. This means that the distribution is the same for the floods of all magnitudes—quite a surprising result.

The writer agrees with the statement of the author (see heading "The Normal Duration Curve") that "a duration curve will be a straight line if the logarithm of variate follows the normal probability curve" if plotted on the logarithmic probability paper. This is the property of the paper; but the writer does not understand why the author advocates the computation of the "plotting positions" by a method which in no way conforms with the normal probability curve, a method that must be abandoned in favor of Eq. 10, when computations become too laborious.

If the curve represents the variations of the position of medians for the floods of various degrees of magnitude, some way should be found to determine a maximum probable deviation in order to get the value of the probable maximum flood. Unfortunately, the writer was unable to discover in the paper any indication of it.

Mr. Beard is to be complimented on his attempt to avoid arcana of mathematics in trying to find a simple way to form an advance judgment regarding the size of the design flood in connection with the expected age of the structure designed to control it.

RALPH W. POWELL,[34] M. AM. SOC. C. E.[34a]—It seems very desirable that references be included in the discussion to other recent publications on this subject, such as those of Prof. E. J. Gumbel[35] and Bradford F. Kimball,[36] and to a type of plotting paper[37] for flood frequency studies based on the former.

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[34] Associate Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

[34a] Received by the Secretary January 22, 1943.

The distribution curve for stream flow is very much skewed. Although it is true that the distribution curve of the logarithms of the flow is much nearer normal, and, therefore, that the lines in Fig. 4 are approximately straight, the writer feels that it is better to use a ruling based on a distribution which more nearly fits the facts. As to the "plotting positions" it would seem that those recommended by the author are not enough different from those used by Mr. Foster,[38] the late Allen Hazen,[39] M. Am. Soc. C. E., *et al.*, to be worth the trouble of computing them, especially since the theoretical basis seems open to question. As a matter of fact, plotting at the lower edge of the band instead of the middle seems to the writer to have much to recommend it. As he understands it, this gives a "frequency" corresponding to Fuller's method and the "California method"—what Mr. Horton calls the "recurrence interval" rather than the "exceedance interval."

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[35] "The Return Period of Flood Flows," by E. J. Gumbel, *Annals of Mathematical Statistics*, Vol. XII (1941), pp. 163–190; "Probability-Interpretation of the Observed Return-Periods of Floods," by E. J. Gumbel, *Transactions*, Am. Geophysical Union, 1941, pp. 836–850; and "Statistical Control-Curves for Flood Discharges," *ibid.*, 1942, pp. 489–509.

[36] "Limited Type of Primary Probability Distribution Applied to Annual Maximum Flood Flows," by Bradford F. Kimball, *Annals of Mathematical Statistics*, Vol. XIII (1942), pp. 318–325.

[37] "A Simple Method of Estimating Flood Frequency," by Ralph W. Powell, *Civil Engineering*, February, 1943, p. 105.

[38] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1217.

[39] "Flood Flows," by Allen Hazen, John Wiley & Sons, Inc., 1930, p. 13.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

ENTRAINMENT OF AIR IN FLOWING WATER
A SYMPOSIUM

Discussion

BY KARL R. KENNISON, M. AM. SOC. C. E.

KARL R. KENNISON,[25] M. AM. SOC. C. E.[25a]—The experiments reported in the paper by Messrs. Kalinske and Robertson throw considerable light on the behavior of water in eliminating air pockets from the high points in pipe lines, and the authors are to be congratulated on the results of their work. From a study of these results the writer believes that the action of flowing water inside a pipe, particularly in the removal of air by successive hydraulic jumps, can be best shown and most easily understood in a comprehensive diagram such as Fig. 19, which shows the condition for any diameter D. The vertical scale represents depth of water in the pipe as shown at the left, and the horizontal scale, the slope of the pipe expressed as the sine of the angle of slope. For convenience of presentation, a cube-root scale has been used in plotting the slopes in order to open up the spacing at the flatter ones.

The additional vertical scale at the extreme right is a convenient criterion of the critical nature of the flow at any depth; that is, its nearness to the critical discharge at which the Froude number, **F**, or $\frac{V_1}{\sqrt{g\, y_e}}$ in the Symposium nomenclature, equals 1. This scale of relation between depth, rate of discharge, and Froude number is independent of the slope or smoothness of the pipe. In considering the capability of the water to remove air, this criterion is particularly important for showing when the flow is subject to surface waves of considerable height. As is well known, such waves can occur very easily, in any kind of channel, where $\mathbf{F} = 1$, as the condition of critical discharge is approached. The reason is that it takes only a very slight disturbance of the pressure-depth relationship—easily disturbed at this unstable stage—to preserve the same total energy with wide variations in the depth. For example, referring to this right-hand scale, if the rate of discharge is 3 (multiplied by

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NOTE.—This Symposium was published in September, 1942, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: January, 1943, by Warren DeLapp, Jun. Am. Soc. C. E.

[25] Chf. Engr., Met. Dist. Water Supply Comm., Commonwealth of Massachusetts, Boston, Mass.

[25a] Received by the Secretary January 8, 1943.

$D^{2.5}$), whenever the pipe is flowing about three-fourths full the Froude number is 1. The same is true for a discharge of 3.5 flowing 0.8 full. In any kind of channel, waves once started under this condition would tend to persist, but inside a circular pipe the rapid and accelerated narrowing of the width as the depth increases, and even the effect of surface tension, tend to trap large air bubbles. Hence, regardless of the slope of the pipe, one would expect a discharge as great as this relative to the pipe diameter to have little difficulty in trapping and removing large quantities of air.

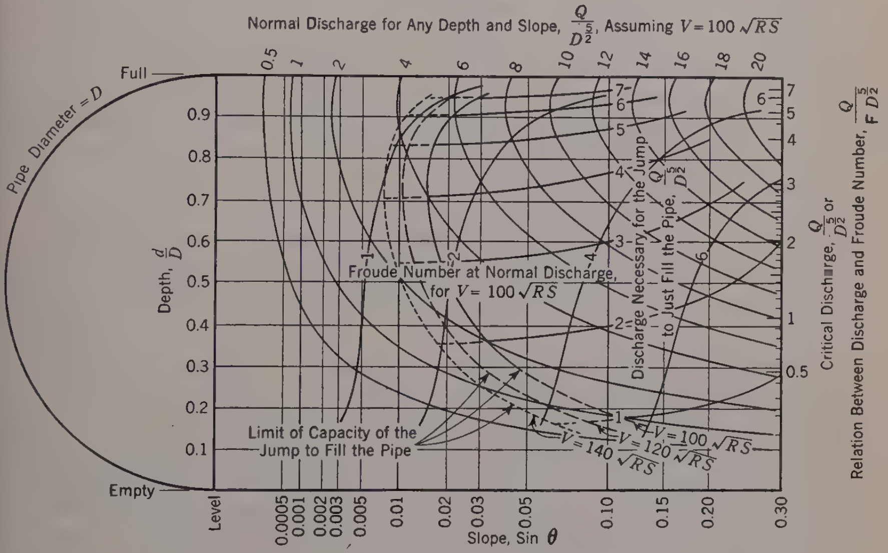

FIG. 19.—HYDRAULIC JUMP INSIDE A PIPE

The effect of slope is indicated by the lines running diagonally across the chart in Fig. 19, and identified by the scale at the top which shows the normal discharge for any depth and slope. The latter are computed for a value of the coefficient, in the formula $V = C\sqrt{R\,S}$, of 100. For smoother pipe, the lines would be moved a little to the left, and vice versa. As air accumulates for any cause at the high point in a pipe line, the surface as the water flows away from this high point cannot drop lower than is indicated by these lines. Introduction of more air merely increases the length of the bubble. For example, if the pipe is on a slope of 0.03 and the rate of discharge is 4 $D^{2.5}$, the chart indicates that, if $C = 100$, the depth cannot decrease to less than 0.55 D.

Consider next what can happen so far as the hydraulic jump is concerned. Messrs. Kalinske and Robertson have obtained considerable information as to the capability of the hydraulic jump to pump air through itself and into the next bubble. The fact that the air cannot escape makes it necessary to include in the formula for the hydraulic jump in a closed pipe a factor representing the amount of the air pumped and the extent to which it decreases the specific gravity of the water. In evaluating the terms of momentum Eq. 35, the authors have omitted from Eq. 36 the factor β, or relation between air and water quantities. In considering the case of the jump barely filling the pipe, the writer has considered the pressure, that is the air pressure in the bubble, the same before as after the jump and has considered that the total pressure on the full pipe section can be represented by the formula

$$\mathbf{F}_2 = \frac{A_p\,\gamma\,D\cos\theta}{2\,(1+\beta)} \qquad (44)$$

The writer has used this form together with the values of β shown in Fig. 13 in solving the jump formula.

In computing the characteristics of the hydraulic jump capable of just filling the pipe, the writer has made an approximation (which, from a study of the situation, seems sufficiently accurate for practical purposes) that the length of the jump is twice its height and, using the Symposium nomenclature, the loss of momentum (divided by D^3) is

$$\mathbf{F}_2 - \mathbf{F}_1 - W\sin\theta = \frac{A_p\,\gamma\,D\cos\theta}{2\,(1+\beta)} - A_1\,\gamma\,d_1\cos\theta - \frac{2\,(D-y_1)\,(\text{average area})\,\gamma\sin\theta}{1+\beta} \qquad (45)$$

The reason for taking length of jump equal to 2 ($D - y_1$) is to be on the safe side in making an air seal against the top of the pipe. Ordinarily, for example, to insure drowning out a jet from a spillway gate (rectangular channel), the writer would make the length three times the height. Note that the use of this multiplier 2 in the formula for the hydraulic jump in a sloping rectangular channel gives a value of the depth after the jump approximately 97% of that for the multiplier 3 on a slope of 0.05, and about 93% on a slope of 0.10.

The results are plotted in the series of almost horizontal curves running across Fig. 19, but a little higher at the right and identified by the vertical scale in the body of the diagram. For example, if the slope of the pipe equals 0.03 and the water is flowing at a depth of 0.55 D, it cannot jump to the top of the pipe unless the quantity flowing is at least 3 (multiplied by $D^{2.5}$). Thus, the intersection of the two sets of curves marks the limit of the capacity of the hydraulic jump to fill the pipe and hence to trap any air and carry it along. This limit is indicated by a line in Fig. 19 drawn through the intersections; and for convenience other similar lines are drawn for the case of somewhat smoother pipe; namely, three limiting lines for C = 100, 120, and 140. These show not only the inability of any jump whatever to form below a certain slope, but also the fact that, for a slope in the vicinity of 0.01, the depth at which a jump can form tends to be confined to a limited range, about two-thirds to three-fourths depth. These curves also show that, under no possible conditions of depth and rate of flow, can a hydraulic jump form inside a pipe if the slope of the pipe is materially less than 0.01.

Fig. 14 appears to be merely an expression of this capability of the jump to fill the pipe. For the low depths, it shows discharges so great that they must be produced artificially. Other curves in Fig. 19 running almost vertically, with their tops bending to the right, show the value of the Froude number, **F**, for the condition of normal discharge at any depth and slope. They are computed for a coefficient of roughness, C, equal to 100, and, as in the case of the lines for normal discharge, they would be moved somewhat to the left in case of a smoother coefficient, and vice versa. For example, these curves show that for any condition of normal discharge throughout a wide range of depths (say, from 0.3 D to 0.7 D), there can be no hydraulic jump in any event unless the slope is as much as 0.0045 and 0.007, respectively—a little less for the smoother pipe. Even though, at a slope somewhat greater than these, a jump does form, it cannot reach the top of the pipe unless the slope is enough greater to fall inside the lines marked "limit of ability of the jump to fill the pipe."

From a study of these and other laboratory test results, in spite of the theoretic considerations summarized in Fig. 19, there seems to be a limiting minimum depth beyond which the jump cannot fill the pipe, for the reason that the approaching jet of water is confined laterally by the shape of the pipe invert. In other words, to fill the pipe requires a much higher rate of discharge than the momentum equation of the jump would indicate. This is shown in Fig. 19 by a lower limit at a depth of about 0.25 D to 0.3 D. The additional loss of energy would appear to be in the piling up at the sides in a manner entirely different from that in a rectangular channel. Other than the important trapping of air bubbles due to surface waves lapping the top of the pipe as described herein for rates of discharge equal to 3 $D^{2.5}$ or 3.5 $D^{2.5}$, Fig. 19 indicates that in a very smooth pipe a rate of discharge only slightly more than 1 $D^{2.5}$, if the slope is at least as steep as 0.02, could successfully pump air from one large bubble to another with a drop in each bubble to a depth of about 0.3 D.

Fig. 19 also indicates that in a case of this kind, where each jump barely makes a seal at the top of the pipe, as soon as sufficient air is removed to decrease the thickness of the air bubble, the discharge ceases to be sufficient to jump to the top of the pipe, with the result that the seal is broken. Accordingly, until further data are available from tests, the writer concludes that (a point not emphasized by Messrs. Kalinske and Robertson) the hydraulic jump alone cannot be as effective in cutting down the head loss in a downward sloping stretch of pipe line as is the trapping of large bubbles by surface waves. In either case the air pressure in each individual bubble is greater than that in the one behind it, the difference being measured by the difference between the slope of the pipe and that of the hydraulic gradient. If this is not the case, then there is no saving of head loss and no point in designing to this end. The case of one long, continuous air bubble from the top to the bottom of the slope is the worst possible condition of air binding, because there is no difference between the slope of the pipe and that of the hydraulic gradient. In such a case all the air that reaches the high point of the line flows out at the bottom of the slope without the hydraulic jump or any other water action lending any material assistance in cutting down the head loss.

The approach of Messrs. Kalinske and Robertson to the problem has been to analyze the behavior of the hydraulic jump in the pipe and its power to pump air. Nevertheless, one should not lose sight of the fact that the ultimate aim of the designer in dealing with these matters is not just to remove air, but to remove it so as to make the most effective saving of head, or to prevent air binding. It would be most useful to have a graph showing the relationship between (1) the slope of the pipe compared with the slope of the hydraulic gradient, (2) the discharge compared with the diameter, or $\frac{Q}{D^{2.5}}$, and (3) the relative quantity of air removed.

The part played by the hydraulic jump in such removal has been most effectively analyzed by Messrs. Kalinske and Robertson.

Correction for *Transactions*: In September *Proceedings*, 1942, page 1145, Fig. 13, change "α" to "β," also, change "0" to "0.01" and "0.01" to "0.015" on the ordinate scale; in Figs. 12(*c*) and 14, change "y_p" to "D"; in Fig. 14, change "$\sqrt{g\,h}$" to "$\sqrt{g\,y_e}$"; and in Fig. 16, page 1149, change "β" to "θ" (Eq. 36, as originally published, is correct.)

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

THE HYDRAULIC JUMP IN SLOPING CHANNELS

Discussion

BY MESSRS. JOE W. JOHNSON, AND KARL R. KENNISON

JOE W. JOHNSON,[9] ASSOC. M. AM. SOC. C. E.[9a]—The problem of the hydraulic jump in a sloping channel is discussed in detail in this paper. The author has presented a formula which is applicable in most practical problems. The experimental coefficient in this formula has been determined over a fairly wide range of the kinetic flow factor.

From experiments in the hydraulic laboratory of the University of California, B. D. Rindlaub[10] developed a relationship for calculating the principal dimensions of a jump in sloping channels. The results of these experiments are of interest in connection with Mr. Kindsvater's paper.

The experiments were conducted in a glass-wall channel, 6 in. wide and 3 ft deep, in which a sloping bed could be set at various angles between 0° and 30°. The flume was equipped with an orifice for flow measurement, a forebay, a tail gate, and point gages for the determination of surface elevations. Most of the experiments were conducted with the bottom set at the angle $\alpha = 12°$; however, experiments also were made with slopes of 8°, 24°, and 30°.

In analyzing the experimental results, Mr. Rindlaub assumed the following equation to describe the jump in a sloping channel:

$$\frac{Q\,\gamma}{g}\left(V_1 \cos\alpha - V_2\right) = \frac{b\,\gamma}{2}\left(d^2{}_1 - d^2{}_2\right) + W \sin\alpha \cos\alpha \ldots\ldots.(25)$$

in which W is the weight of the water in the jump. As in the case of the jump in a horizontal channel, the variables in this equation may be separated into dimensionless groups. If l represents the length along the floor from the toe of the jump to the point of change in grade $\left(l = \frac{L_s}{\cos\alpha} \text{ in Fig. 7 of the paper}\right)$, and d_1, d_2, α, and Q are as defined by the author, the following dimensionless

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NOTE.—This paper by Carl E. Kindsvater, Jun. Am. Soc. C. E., was published in November, 1942, *Proceedings*.

[9] Asst. Prof., Dept. of Mech. Eng., Univ. of California, Berkeley, Calif.

[9a] Received by the Secretary December 28, 1942.

[10] "The Hydraulic Jump in Sloping Channels," by Bruce D. Rindlaub (unpublished), a thesis presented to the University of California, Berkeley, Calif., in 1935, in partial fulfilment of the requirements for the degree of Master of Science.

groups may be written:

(*a*)..............α

(*b*)..............$\dfrac{d_2}{d_1}$

(*c*)..............$\lambda_1 = \dfrac{Q^2}{g\, d^3_1}$ and $\lambda_2 = \dfrac{Q^2}{g\, d^3_2}$

and

(*d*)..............$\dfrac{l}{d_1}$

The method of summarizing the experimental data by the use of these dimensionless groups is illustrated by Fig. 10. For the data taken with a bed slope of 12°, a plot of the ratio $\frac{d_2}{d_1}$ against the kinetic flow factor, λ_1, is shown for various values of $\frac{l}{d_1}$. The plotted points show a fairly consistent variation and, therefore, permit curves of constant $\frac{l}{d_1}$ to be constructed. For comparative purposes the relationship between $\frac{d_2}{d_1}$ and λ_1 for a jump in a horizontal

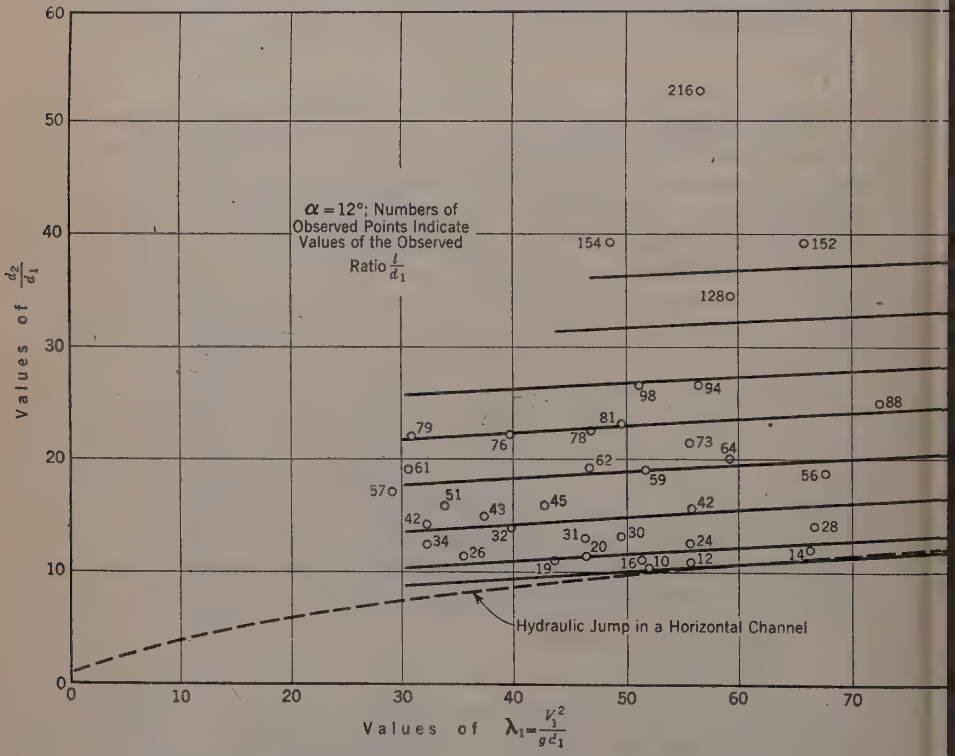
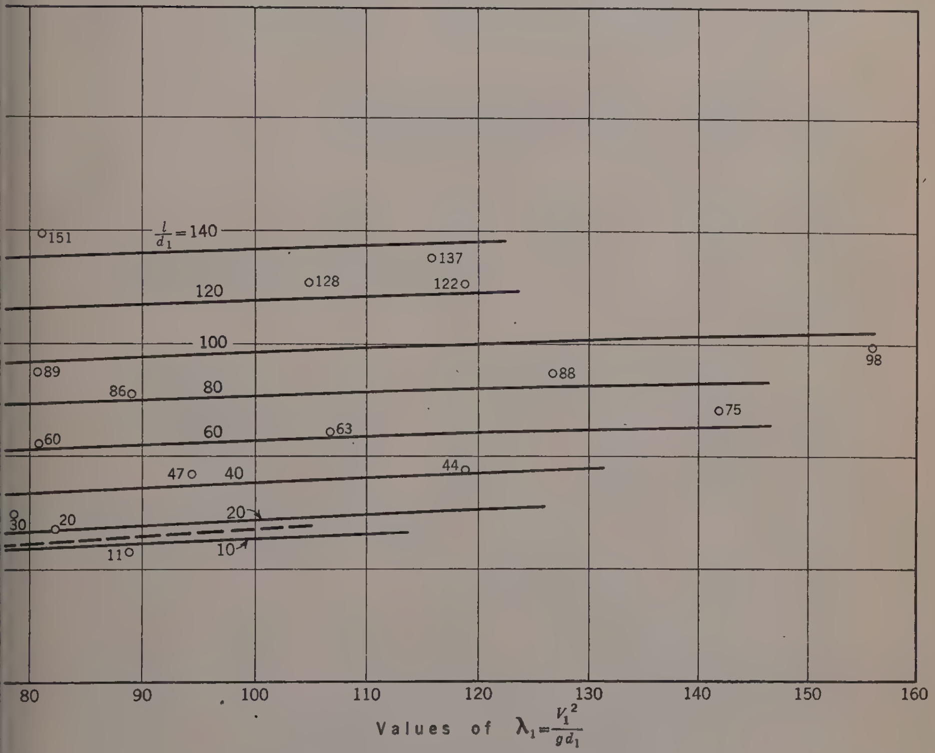


FIG.

channel also is shown in Fig. 10. Data taken on channels with slopes of 8°, 24°, and 30° gave curves similar to those in Fig. 10. In the experiments the value of l was varied between 0 and 4.11 ft, d_1 between 0.013 and 0.224 ft, d_2 between 0.25 and 1.18 ft, and Q between 0.056 and 0.670 cu ft per sec. All four of the author's classifications of the jump in sloping channels, consequently, were included in the experiments.

The characteristics of a jump on a particular slope may be computed from plots similar to that illustrated in Fig. 10. Thus, by knowing the values of d_1 and Q, and by assuming a value of l, the values of $\frac{l}{d_1}$ and λ_1 can be computed. The plot of $\frac{d_2}{d_1}$ versus λ_1 for the particular slope can then be entered and the value of $\frac{d_2}{d_1}$ corresponding to the computed values of $\frac{l}{d_1}$ and λ_1 determined. Finally, the value of d_2 can be calculated from $\frac{d_2}{d_1}$.



10

This procedure is similar to the solution of Eq. 24 as described by the author. Mr. Kindsvater's generalized method of analysis, perhaps, is more practicable because curves of $\frac{d_2}{d_1}$ versus λ_1 (similar to Fig. 10) for the particular slope are not required. The author is to be complimented on his presentation of a simple and convenient method of computation that is suitable for most design problems.

KARL R. KENNISON,[11] M. AM. SOC. C. E.[11a]—The author has done an excellent job in summarizing the available data in usable form. The subject of the paper seems to be somewhat of a misnomer, since it does not deal with continuous sloping channels, referred to as "a fifth case," but rather with the extent to which a sloping jet striking a horizontal apron can be backed up the slope. The results are very much worth while, particularly because they contain empirical factors resulting from important experimental work.

The computations involved in the theoretical deductions of the formulas seem to be more complicated than warranted by the actual uncertainties in flows of this type. For example, these problems generally deal with a condition where the neglected friction loss is an important item. In the case of any formula for the jump in sloping channels, it should be borne in mind that this fact is certain to vitiate the results, particularly in the case of low Froude numbers.

The writer believes that the formula for the hydraulic jump in a sloping channel (made general to include the angle of slope, if any, of the channel bottom) can be simplified, both in the derivation and in the result, by equating the momenta before and after the jump, considering the forces parallel to the channel bottom, and by expressing the height of the jump as a function of its length. Referring to Fig. 11, the momentum equation is as follows, using the author's nomenclature:

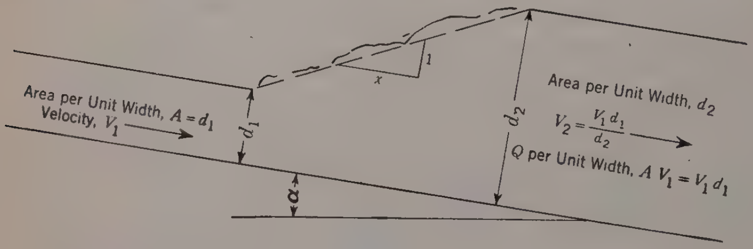


FIG. 11

$$\frac{Q\,\gamma\,V_1}{g} - \frac{Q\,\gamma\,V_2}{g} = \frac{\gamma\,d_2^2\cos\alpha}{2} - \frac{\gamma\,d_1^2\cos\alpha}{2} - x\,(d_2 - d_1)\left(\frac{d_2 + d_1}{2}\right)\gamma\sin\alpha \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

This becomes

$$d_2 = \frac{d_1}{2}\left(\sqrt{\frac{8\,\lambda}{\cos\alpha - x\sin\alpha} + 1} - 1\right) \ldots\ldots\ldots\ldots (27a)$$

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[11] Chf. Engr., Met. Dist. Water Supply Comm., Commonwealth of Massachusetts, Boston, Mass.

[11a] Received by the Secretary January 8, 1943.

Instead of expressing the depth after the jump as the height normal to the bottom of the slope, consider the author's case of impinging on a horizontal apron and take d_2 vertical; then Eq. 26 becomes

$$d_2 = \frac{d_1}{2 \cos \alpha} \left(\sqrt{\frac{8 \lambda}{\cos \alpha - x \sin \alpha} + 1} - 1 \right) \ldots\ldots\ldots\ldots (27b)$$

which is to be compared with Eq. 14.

It is sufficiently accurate for practical purposes to assume that the length of the jump is three times its height, or that $x = 3$. Actually this simpler formula with $x = 3$, will give the same result as the author's, for a kinetic flow factor, λ, in the vicinity of 50. The choice of the value to assign to the multiplier, x, makes surprisingly little difference in the result. Using $x = 2$ gives a resulting value of d_2 which is 97% of that using $x = 3$, for a slope of 0.05; and 93%, for a slope of 0.10.

Corrections for *Transactions:* In November, 1942, *Proceedings*, page 1475, line 25, change "$(V_1)^{\frac{1}{2}}$" to "$(V_1)^2$"; and, on page 1478, in Fig. 5(a), draw arrow from the equation to the upper shaded area.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

CONFORMITY BETWEEN MODEL AND PROTOTYPE

A SYMPOSIUM

Discussion

BY V. L. STREETER, ASSOC. M. AM. SOC. C. E.

V. L. STREETER,[18] ASSOC. M. AM. SOC. C. E.[18a]—Those in charge of hydraulic laboratories are inclined to look to model studies for the answer to all hydraulic questions, when the application of fundamental hydraulic design principles will frequently give more reliable information at a small fraction of the cost. In many cases, a more detailed and searching analysis of the problem before the model is constructed would show that a complete model study of all hydraulic features is unnecessary. To be specific, in the case of pressure and discharge tests on the 102-in. river outlets described by Messrs. Warnock and Dewey, a more reliable discharge may be obtained by computation than by scaling up the model discharge. Although Froude's number must be used to scale the discharge, the value of friction factor, which depends on the Reynolds number, cannot be determined from the model.

The writer, while working in the hydraulic laboratory of the U. S. Bureau of Reclamation, prepared a prototype discharge curve for the intermediate outlets before the model was constructed. For values of the head, H, greater than 40 ft, one outlet has the discharge

$$Q = 340\ H^{0.5} \qquad (18)$$

in which, supplementing the Symposium nomenclature, H is the total head above the center of the exit, in feet. Eq. 18 is based on:

(1) A loss at the entrance and gate section of one tenth the velocity head in the pipe;

(2) A friction factor $f = 0.0073$ for the 225 ft of pipe; and

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NOTE.—This Symposium was published in October, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1942, by A. E. Niederhoff, Assoc. M. Am. Soc. C. E.; and January, 1943, by Messrs. C. I. Grimm, and Joe W. Johnson.

[18] Associate Prof., Hydraulics, Illinois Inst. of Technology, Chicago, Ill.

[18a] Received by the Secretary January 4, 1943.

(3) A loss of six hundredths of the velocity head for elbow and converging section. The friction factor was determined by a consideration of smooth pipe friction factors for the same Reynolds numbers, as given by the Níkuradse (59)[18b] formula. The curve resulting from this discharge equation is plotted in Fig. 66 with those given in Fig. 13. This computed discharge curve seems

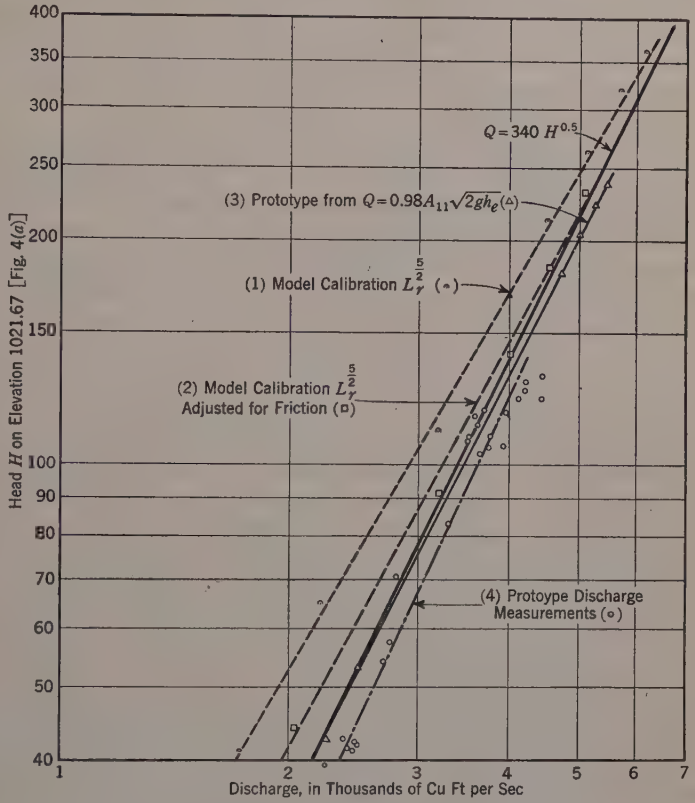

FIG. 66.—COMPARISON OF DISCHARGES (SEE FIG. 13)

more reliable than the curve through the points obtained from the prototype discharge measurements. Curve 4, representing the prototype discharge measurements, has a loss of head proportional to $V^{2.13}$, in which V is the average velocity in the 102-in. outlet. It is significant that Messrs. Warnock and Dewey did not use the discharge measurements as a basis for computing the prototype friction factor.

The prototype friction factors as shown in Fig. 6 may easily be 25% too high. To determine accurately the value of f for a pipe with small length-diameter ratio, velocity distributions should be obtained at each end of the test section. At the downstream end of the bellmouth the velocity distribution would be practically uniform, but about fifty diameters farther downstream the velocity would assume its characteristic distribution for the particular Reynolds number. The value of the kinetic energy correction factor for a friction factor 0.0073 would be about 1.02 (60). For a head of 150 ft, the velocity head is about 83.6 ft. Neglecting the kinetic energy correction factor could change the head loss by two hundredths of the velocity head, or 1.67 ft. The head loss due to friction in 40 ft of outlet should be about 2.87 ft. Hence a change in kinetic energy correction factor of 0.011 in the reach would change the friction factor by 25%.

[18b] Numerals in parentheses, thus: (59), refer to corresponding items in the Bibliography, which is appended to the Symposium as a separate unit, and to the end of discussion in this issue.

In many cases the best method of predicting prototype performance is first to formulate analytical relationships for model performance. When relationships are established so that model performance may be predicted and verified by tests, the analytical relationships may be altered to fit the prototype. The discharge curve for the outlet works is an example of this method. Using the best values of f, and minor losses, the discharge for the model may be obtained in equation form, and the various losses may then be adjusted to agree with model experimental data. When this is accomplished, the value of f should be changed to that anticipated in the prototype. As the minor losses are more dependent on Froude's number, they should not require alteration. This method permits an evaluation of model performance in terms of the prototype when the Reynolds number effects are important.

The authors' explanation of the discrepancy in pressures between the model and prototype—that is, due to unstable flow—is not clear (heading: "Examples of Quantitative Comparisons: Pressure Drop in Elbow and Cone"). For a scale of 1 : 17 the Reynolds number for the model would be about one seventieth as much as for the prototype. For the outlet model the Reynolds number would be between 300,000 and 1,000,000—a range in which there is no reason to expect unstable flow.

The observed pressure distributions through the elbow and cone, as given in Fig. 5, do not appear susceptible to analysis. Forgetting the observed results, for the time being, the fundamental hydraulic relationships will be examined. If an elemental volume of water is moving along a curved path of radius, r, with velocity, v, the difference in pressure heads, $\frac{dp}{w}$, on the two sides of the element normal to the radius of curvature, must be $\frac{v^2}{g\,r}\,dr$, in which dr is the radial distance between the two sides, and g the acceleration of gravity. For a finite layer of water moving in a curved path, the difference in pressure head is given by

$$\frac{\Delta p}{w} = \frac{1}{g}\int \frac{v^2}{r}\,dr \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

which is integrated between the outer and inner filaments of flow. If the values of v and r are known for each filament, this relation will give the proper difference in pressure head, assuming the path of integration to be in a horizontal plane. Unfortunately these relationships are not known. Assuming the center of curvature of all elements on a line through the center of curvature of the elbow perpendicular to the plane of the elbow, three different assumptions regarding velocity distributions will be made. First, if the velocity is everywhere constant in magnitude, the pressure head drop, $\frac{\Delta p}{w}$, from crown to invert for the intermediate outlets, in feet of water would be

$$\frac{\Delta p}{w} = 0.00000206\, Q^2 - \Delta z \qquad (20a)$$

in which Δz is the difference in elevation of crown and invert along a radial line from the center of curvature. The term Δz corrects the formula for the case in which the filaments are not in a horizontal plane. If, instead of the foregoing velocity assumption, it is assumed that the magnitude of the velocity varies inversely as the distance from the center of curvature, as with the "free vortex," the pressure head drop in feet of water becomes

$$\frac{\Delta p}{w} = 0.00000216\, Q^2 - \Delta z \qquad (20b)$$

Finally, assuming that the velocity magnitude varies directly as the distance from center of curvature, as with the "forced vortex," the pressure head drop becomes

$$\frac{\Delta p}{w} = 0.00000205\, Q^2 - \Delta z \qquad (20c)$$

It is surprising how well Eqs. 20 agree. One is led to think the velocity distribution is not too important. Approximately the pressure drop is found to vary inversely with the radius of curvature for each assumption regarding velocity distribution. If the elbow turned through an angle of, say, 180°, there would be no question as to whether the center of curvature used was substantially correct. For an angle 41° 19′, however, the radius of curvature of the filaments may for the most part be greater than that of the elbow. Using the results of the model studies (Fig. 5) to determine this radius for the assumption of constant velocity, the radius of curvature at the center line is found to be 52.8 ft. This is based on an average of eight values taken from the two higher model heads shown in Fig. 5 for distances from the axis of 195, 200, 205, and 210 ft. The spread was from 48 ft to 60.5 ft. Using the value 52.8 ft, the formula becomes

$$\frac{\Delta p}{w} = 0.00000155\, Q^2 - \Delta z \qquad (20d)$$

Checking Eq. 20*d* against the prototype measurements in Fig. 5 corresponding to those for the model, the pressure head drops are found to agree on the average within 2.4%, with a spread from − 11% to + 4.5%.

This is a method, therefore, of predicting the difference in pressure between prototype crown and invert from model studies. From considerations of equilibrium in the elbow, the pressure rise at the crown may be shown to equal the pressure drop at the invert, assuming a "forced vortex" velocity distribution. Approximately the same results will be obtained for the other two assumptions. The pressure at the center line would not change because of the centrifugal forces set up in the elbow. The poor agreement shown in Fig. 5 would appear due to errors in model piezometer pressures.

This discussion has been submitted in order to emphasize the importance of careful analytical study of hydraulic design features before construction of a model and the continued analytical attitude during model testing and evaluation of results.

Bibliography.—

(59) **"Gesetzmässigkeiten der turbulenten strömung in glatten röhren,"** by J. Nikuradse, Verein Deutscher Ingenieur, Forschungsheft, No. 356, Berlin, Germany, 1932.

(60) **"The Kinetic Energy and Momentum Correction Factors for Pipes and Open Channels of Great Width,"** by Victor L. Streeter, *Civil Engineering*, April, 1942, p. 212.

(61) **Technical Report for 1941**, Central Board of Irrigation, *Publication No. 27*, India: (*a*) "A Comparison of the Results of Model Experiments and Observations on the Prototype," by Harbans Lal Uppal, p. 64. (*b*) "Working of the Armoured Spurs on the Beas River during the First Year and Comparisons with the Results Obtained on the Model," by J. L. Roy, p. 68. (*c*) Discussion of Model-Prototype Relations, Research Committee, p. 69.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

DISTRIBUTION GRAPHS OF SUSPENDED-MATTER CONCENTRATION

Discussion

BY H. A. EINSTEIN, ASSOC. M. AM. SOC. C. E.

H. A. EINSTEIN,[3] ASSOC. M. AM. SOC. C. E.[3a]—A method is outlined in this paper for obtaining a dependable record of suspended load with only a fraction of the number of samples required without its use. This procedure reduces the rather high cost for any individual station considerably, and it will prove to be of greatest significance for any future suspended-load sampling program. The paper is theoretically important because it shows, more clearly than ever has been done before, the necessity of dividing the sediment load of a stream into bed-material load and wash load. The limiting size between the two is to be determined for any station before load measurements are started, because it divides the total load into two fully independent parts that follow strictly separate laws of transportation and, therefore, are to be determined according to entirely different rules.

The concept "time-of-lead" is introduced by Professor Johnson for the time interval between maximum concentration and maximum discharge, and it is explained by the rapidly decreasing availability of fine sediment in the field during a rain. In watersheds as small as those studied by the author, this original "time-of-lead" as created in the field can be found in any section of the watershed, since mere addition of sediment loads and discharges will not materially affect this time interval. In large streams, however, another influence that tends to change this time interval becomes effective. The sediment in suspension travels with the average speed of the water, or somewhat slower (especially the coarse particles, which are concentrated near the bed where the velocities are less than average). Changes of discharge travel with wave velocity, which is considerably higher than the flow velocity in most streams. This fact sometimes enables the maximum of discharge to overtake the maximum concentration. This phenomenon occurs most frequently in

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NOTE.—This paper by Joe W. Johnson, Assoc. M. Am. Soc. C. E., was published in October, 1942, *Proceedings*.

[3] Hydr. Engr., SCS, U. S. Dept. of Agriculture, Greenville, S. C.

[3a] Received by the Secretary January 15, 1943.

parts of streams below long stretches without tributaries. Probably the best example has been published by Alfred Dale Lewis under the title "Silt Observations of the River Tigris,"[4] in which he describes a solitary flood of the River Tigris at Amara, Iraq, in January, 1918. The flood peak occurred on January 12, with a silt concentration equal to that before the flood. Only after the peak had passed did the concentration start to increase; and it reached its maximum on January 15, three days after the peak of the flood. Such extreme cases are rare, but the effect of the difference between the velocities of the sediment and of the flood peak is common especially in larger rivers.

There is another reason why the logic of applying the unit-graph method to large rivers is problematic. This method basically assumes a normal distribution over the entire watershed of the precipitation that causes the rise. The larger the watershed, the smaller will be the probability that the distribution of the precipitation during any one storm occurs according to the normal pattern. Even in a watershed as small as that of Enoree River (65 sq miles), mentioned by the author, storms will be found that deviate widely from the normal. This fact can be observed very readily in this river, because its normal flood has two distinct peaks, a smaller one from rain in the lower part of the watershed and a somewhat higher second peak from the rain in the upper part of the basin. These two sections are distinctly separated by a stretch of the river without major tributaries. Some few storms occurred in this watershed with precipitation in either the lower or the upper part of the basin only. The resulting hydrographs showed only one peak, and, therefore, could not be handled by the unit-graph method. Cases of this kind are rare in small physiographically uniform watersheds but become more and more frequent the larger and the more complex the watershed under consideration.

In the practical application of the unit-graph method as proposed by the author, it is important to bear in mind, therefore, that although its application to streams with small and uniform watersheds has been proved to give good results with a minimum of cost, its applicability to streams with large and complex watersheds seems questionable and for this reason needs further study.

Correction for *Transactions:* October, 1942, *Proceedings*, page 1390, to the caption of Fig. 3 add "(Time Interval of One-Half Hour Used in Preparation of Graphs)."

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[4] Minutes of *Proceedings*, Institution of Civ. Engrs., Vol. 212, 1921, pp. 393–399.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

DEWATERING, INCINERATION, AND USE OF SEWAGE SLUDGE

A SYMPOSIUM

Discussion

By A. L. Genter, M. Am. Soc. C. E.

A. L. Genter,[8] M. Am. Soc. C. E.[8a]—Mr. Schroepfer's contribution to this subject is excellent in that it shows how far the economics of raw sludge dewatering can be pushed with engineering acumen and operating skill. However, as he states, it relates strictly to raw sludge handling in a specific plant; that is (see heading, "Effect of Filter Rate on Cost of Filtration"),

> "It is based on the conditions (type of sludge, chemical costs, etc.) existent at this plant [Minneapolis-St. Paul]. A comparable analysis can, and should, be made by other operating plants, based on the conditions obtaining there, to determine (*a*) the economics of operation of present installation, (*b*) possibilities of effecting economies by the enlargement of present facilities at some future date, and (*c*) economical rates for different types of sludges for the benefit of designers of future plants."

The question of comparison with other plants is not so much how other vacuum filter practice fits into the picture of Minneapolis-St. Paul, but how the practice at this plant fits into the economics of another type of plant where the sludge solids are further mineralized—that is, the bulk of the sludge is reduced by sludge digestion, and the fouled natural sludge moisture is removed by elutriation prior to filtration. This procedure is essentially the reverse of practice at Minneapolis-St. Paul where solids reduction follows filtration. The First Progress Report of the Committee of the Sanitary Engineering Division on Sewerage and Sewage Treatment (27)[8b] shows that sludge elutriation is practiced for some 3,000,000 population in various parts of the United States and Canada, namely for a greater total population than at Minneapolis-St. Paul.

Note.—This Symposium was published in January, 1943, *Proceedings*.

[8] Cons. Engr., Baltimore, Md.

[8a] Received by the Secretary January 12, 1943.

[8b] Numerals in parentheses, thus: (27), refer to corresponding items in the Bibliography at the end of the Symposium and at the end of discussion in this issue.

Instead of referring to Mr. Fischer's six-year-old and rather incomplete paper for illustrative purposes of comparison, as Mr. Schroepfer has done (8), the writer deems it more appropriate to compare the 1941 averages at Minneapolis-St. Paul to some of these widely spread typical plants for the same year. The tables and remarks herewith submitted are intended therefore for this purpose.

In the light of what is now known about the chemical conditioning and vacuum filtration of various types of sewage sludges, it should be emphasized at the outset that the raw sludge at Minneapolis-St. Paul is relatively easy to condition with low chemical doses prior to filtration. This can be attributed to (1) its relatively low volatile and high ash content, (2) consequent high concentration of solids, and (3) low alkalinity of the water associated with the sludge solids. If the volatile percentage at Minneapolis-St. Paul were the same as that of the primary raw sludge at the Richmond-Sunset treatment plant in San Francisco, Calif. (82% instead of 62.1%), the annual chemical conditioning costs would be at least 250% greater than those listed by Mr. Schroepfer. Furthermore, such a sludge would be materially thinner, would require more filter area, filter hours, and incineration hours, with a consequent increase in power expended.

The items of volatile-to-ash ratio in the sludge solids, alkalinity of sludge water, and the amount of sludge solids per capita per year control local vacuum filter operating economies in all plants. They are relatively favorable at Minneapolis-St. Paul notwithstanding the facts that the solids per capita are unduly high and the natural tendency in dewatering raw sludge is to increase the vacuum filter equipment. Mr. Schroepfer's record of pushing the economics of sludge dewatering to limits below those of other plants dewatering relatively large amounts of raw sludge is primarily an excellent engineering account of all the economic consequences of the vagaries of raw sludge—(1) "inconsistency of sludge behavior characteristics" (see heading, "Improvements in Filtration as Applied to Minneapolis-St. Paul Project"), (2) frequent washing of filter cloths (every four hours), (3) lime incrustation with the consequent necessity of frequent acid washing of cloths, screens, drums, and filter piping while using analine inhibitor, (4) problem of returning incinerator ash to help mineralize the sludge, thereby increasing the solids to be dewatered, and (5) the use of "a relatively large number of professional engineers and chemists in supervisory positions" (see heading, "Costs of Dewatering and Incineration as Compared to Other Processes of Sludge Disposal: Personnel Required"), together with (6) skilled operators to control all these functions.

With sludge digestion and elutriation, this list of functions and problems cannot help but be in the opposite direction; in other words, there is much less sludge to be handled, greater mineralization of the sludge solids, far less dependence on the vagaries of sludge, far less chemical and power requirements, and greater filter yields. The writer agrees thoroughly with the argument that operating intelligence is a prerequisite to sludge filtration economy in all plants. However, Mr. Schroepfer's following statement (heading, "Costs of Dewatering and Incineration as Compared to Other Processes of Sludge Disposal") needs qualification: "To expect that even the most conscientious operator of a plant in a community of, say, 25,000 people, in which 1 to 2 tons of dry solids must be disposed of daily, can even approach the efficient and controlled operation of a plant handling 50 or more tons daily is erroneous." With equal operating acumen in both cases, this depends principally on the sludge processing methods employed. Undoubtedly, in a larger plant with more equipment and greater operating staff, a greater amount of sludge solids will be disposed of per man-hour of work. However, when it comes to dewatering various types of sludges in vacuum filters, "efficiency" is a relative term. Doing a job efficiently means doing it very well at the least possible cost. Mr. Schroepfer has demonstrated ably that it is difficult and requires "eternal vigilance" to dewater large quantities of raw sludge each day, economically, and that he and his associates are doing this job exceedingly well. However, the application of efficiency to dewatering elutriated digested sludges is far easier from the operator's standpoint and cheaper from the taxpayer's viewpoint. For this reason, the small plant serving about 25,000 population at Annapolis, Md., is included in Table 19.

This plant is under a superintendent devoting his time to a distant sewage pumping station, sewage chlorination, and the entire treatment plant operation without the benefit of chemical control other than that which his time allows and the Maryland State Department of Health requires. At the treatment plant, a helper aids him in operating a small elutriation tank and vacuum filter with ancillary equipment. This work requires only three days weekly and then only 4 or 5 hr daily, without the eternal vigilance and fight against lime incrustation evils common to Minneapolis-St. Paul. The latter plant serves a population about 32 times greater than does the plant at Annapolis, and yet the chemical cost during 1941 at Minneapolis-St. Paul was 187 times greater than at Annapolis (see Table 19(*c*)). Undoubtedly the same operator at this small plant could operate, in the same time schedule and at the same wage, a vacuum filter six times larger (390 sq ft), and handle six times as much elutriated sludge with practically the same ease as at present, thus reducing labor costs per ton of sludge solids to one sixth the present cost. This fact, plus the high power cost (3.5¢ per kw-hr), naturally increases costs per ton of sludge solids dewatered. However, such factors have little to do with efficiency and ease of filter control and operation. The correction of these latter factors primarily starts in the sludge itself; in other words, they depend on the constancy of sludge characteristics or uniformity conditions which produce the chief aid to constancy of operating averages.

Such operating comparisons, as herein offered, ultimately come back to the taxpayer through the annual operating budget, meaning that the essential items controlling sludge dewatering costs should be measured in terms of per capita per year. This is the yardstick adopted in Table 19.

Table 19(*a*) presents the basic data from which most of the information in the remainder of the table is drawn. Sludge dewatering practice in the listed plants is limited to handling primary raw sludge (Minneapolis-St. Paul), digested primary (Buffalo, N. Y.), and elutriated digested primary in the remainder of the plants serving populations of about 800,000 at the District of Columbia (about the same as Minneapolis-St. Paul) down to 25,000 at Annapolis.

TABLE 19.—Comparison of Essential Items That Control Dewatering Costs (1941 Data)

(a) Basic Data

Item No.	Description	Minneapolis-St. Paul, Minn.	Buffalo, N. Y.	District of Columbia	Hartford, Conn.	Springfield, Mass.	Winnipeg, Manitoba, Canada	San Francisco, Calif.	Annapolis, Md.
1	Approximate population	805,000	600,000	800,000	150,000	130,000	255,000	145,000	25,000
2	Total sewage treated (million gal)	39,286.7	51,262	39,568	5,200	5,799	9,572	2,940.6	703.8
3	Suspended solids (ppm)	315	168	180	160	155	329	254	....
4	Percentage removal in plant	74.6	34.8	51	66.5	58.1	54.6	70.5	....
5	Type of sludge dewatered	Raw primary	Digested	Primary digested[a]	Primary digested[a]	Primary digested[a]	Primary digested[a]	Primary digested[a]	Primary digested[a]
	Percentage of Volatile:								
6	Raw	62.1	62.9	66.2	66.3	73.4	69.7	82.0	....
7	Digested	....	47.5	45.1	45.8	45.8	51.1	63.5	51.4
8	Elutriated	....	....	44.4	39.8	48.3	50.5	....	50.0
	Raw Solids:								
9	Tons produced	35,993	18,961	15,000	2,505	3,184	8,769.5	2,748	548
10	Tons dewatered	35,993	....	....	....	....	....	....	....
11	Lb per capita per yr	89.5	63	37.5	33.4	49	69	37.9	43.8
	Digested Solids Dewatered:								
12	Tons	....	10,051	6,870	1,296	946	1,655.2	888	173.2
13	Lb per capita per yr	....	33.5	17.1	17.3	14.6	13	12.3	13.8
	Filter Operation:								
14	Days	351	306	220	156	220	182	197	145
15	Hours	39,919.1	6,320	4,705.1	994	2,277.5	5,407	1,682	710.5
16	Percentage solids in feed	9.35	9.11	6.5	6.8	11.4	10.0	3.15	9.0
	Percentage of Chemical Dose on Solids:								
17	Ferric chloride	1.53	2.93	3.18	2.23	1.63	2.6	4.3	....
18	Lime (Ca O)	3.77	11.21	....	....	....	....	....	....
19	Ferric sulfate	....	....	....	....	....	....	....	2.86
20	Filter yield	3.9	6.77	5.9	7.5	5.0	4.2	5.4	7.5
21	Power cost (cents per kw-hr)	1.163	0.73	0.63	2.32	1.9	0.625	1.1	3.5
22	Fe Cl₃ (dollars per ton)	38.21	31.80	39.00	71.40	72.00	83.60	60	99.6
23	Ferri-floc (dollars per ton)	....	....	....	....	....	....	....	36.10

(b) Total Cost of Sludge Filtration and Sludge Disposal by Incineration or Hauling (Cents per Capita Served by Plant)

Item No.	Description	Minneapolis-St. Paul, Minn.	Buffalo, N. Y.	District of Columbia	Hartford, Conn.	Springfield, Mass.	Winnipeg, Manitoba, Canada	San Francisco, Calif.	Annapolis, Md.
24	Filtration	9.64	....	3.98[b]	4.25[b]	5.7[b]	5.4[b]	7.0[b]	7.2[b]
25	Incineration or disposal	6.04	....	2.61	1.40	3.22	1.62	1.7	0.0
26	Total	15.68	15.63	6.59	5.65	8.92	7.02	8.7	7.2

(c) Chemical Costs for Sludge Filtration

Item No.	Description	Minneapolis-St. Paul, Minn.	Buffalo, N. Y.	District of Columbia	Hartford, Conn.	Springfield, Mass.	Winnipeg, Manitoba, Canada	San Francisco, Calif.	Annapolis, Md.
27	Total spent (dollars per yr)	33,410.70	18,382.43	8,500.00	2,060.90	1,108.00	3,803.74	2,300.00	178.70
	Cost, in Cents per Capita:								
28	Actual	4.15	3.06	1.06	1.37	0.852	1.49	1.59	0.714
	With Fe Cl₃ at—								
29	$38.21 per ton	4.15	3.32	1.04	0.74	0.45	0.68	1.01	0.45[c]
30	$31.80 per ton	3.71	3.06	0.869	0.61	0.38	0.57	0.84	0.38[c]

	(*d*) POWER CONSUMPTION FOR FILTRATION AND FILTER HOURS								
	Power Used:								
31	Total kilowatt-hours annually	751,889	....	302,300[d]	48,750[d]	90,000[d]	96,309[d]	63,000[d]	3,695[d]
32	Kw-hr per 1,000 population per year	934	....	378	325	692	378	434	148
33	Filter hours per 1,000 population per 100 sq ft of filter area per year[e]	248	52.7	29.4	23.2	28.9	31.8	23.2	18.5
	(*e*) LIFE OF FILTER CLOTH AND CONSUMPTION PER THOUSAND POPULATION SERVED								
34	Kind of cloth (CF = canton flannel)	CF	CF	Wool	Wool	Wool	Wool	Wool	CF
35	Acid wash necessary	Yes	Yes	No	No	No	No	No	No
36	Average filter hours per filter[e]	6,653.2	2,106.7	1,176.3	497	1,138.8	2,703.5	1,682	710.5
37	Economic cloth life (hours)	356	211	600	600	800	284	900	142.1
	Dressings:								
38	Total number annually	112	30	8	1.66	3	12	2	5
39	Number per filter	18.7	10	2	0.83	1.42	6	2	5
	Area of Dressings:								
40	Total used annually (sq ft)	56,000	15,000	4,000	581	495	1,800	400	325
41	Square feet per 1,000 population	69.57	25	5	3.9	3.8	7	2.76	13
	(*f*) SERVICE CAPACITIES								
42	Number of filters	6	3	4	2	2	2	1	1
43	Total area (sq ft)	3,000	1,500	2,000	700	330	300	200	65
44	Filter hours per operating day[e]	113.73	20.65	21.39	6.37	10.35	29.71	8.54	4.9
	Population Per Filter Hour Per Operating Day:								
45	Total	7,078.2	29,050	37,401	23,548	12,560	8,583	17,000	5,100
46	Per square foot of area	14.15	58.1	74.7	67.2	76.1	57.2	85	78.5
	(*g*) POPULATION SERVICE WITH ALL INSTALLATIONS OPERATING ON THE MINNEAPOLIS-ST. PAUL TIME TABLE								
	Average Hours per Filter Per Day for:								
47	Actual operating day	18.96	6.88	5.35	3.19	5.18	14.86	8.54	4.9
48	351 operating days	18.96	6.00	3.35	1.42	3.24	7.7	4.79	2.02
49	Increased ratio possible at 18.96 hr per filter daily	None	3.16	5.66	13.35	5.84	2.46	3.96	9.38
	Possible Population:								
50	At the increased ratio[f]	805,000	1,896,000	4,528,000	2,000,000	759,000	627,000	574,000	234,000
51	Per square foot per filter hour at this increase (Item No. 50)	14.15	66.6	119.4	150.7	121.3	110.2	151.4	190

[a] Primary digested and elutriated. [b] In all elutriation installations, the cost of sludge elutriation is included in the filtration costs. [c] Annapolis uses Ferri-floc at $36.10 per ton; but during 1940 used a ferric chloride dose of 1.7% on solids, at a cost of $99.60 per ton. [d] In all elutriation installations, the data include power for all elutriation purposes. [e] For total filter days and hours see Items Nos. 14 and 15, Table 19(*a*). [f] Item No. 49 multiplied by Item No. 1, Table 19(*a*).

The flocculation and primary treatment scheme used at the Richmond-Sunset treatment plant is somewhat similar to that used at Minneapolis-St. Paul, although the primary sludge at the former plant is of much higher volatile content and therefore more difficult to settle, digest, and dewater. The greatest sewage flow is at Buffalo. The flows at Minneapolis-St. Paul and the District of Columbia are practically identical. Both Baltimore, Md., and the District of Columbia treatment plants serve about the same population and are the largest elutriation installations. However, Baltimore treats some of its sewage by sludge activation and the digested sludge contains some digested waste activated as well as digested primary, and the digested sludge is given only single-stage elutriation treatment. Consequently the results for the District of Columbia plant are listed herein. Springfield, Mass., is included with Hartford, Conn., as sludge incineration is used at the former plant and not at the latter.

For 1941 the per capita expenditures at the listed plants for sewage treatment (operation and maintenance, exclusive of pumping and chlorination) were: Minneapolis-St. Paul, 38¢; Buffalo, 47.35¢; District of Columbia, 17.26¢; Hartford, Conn., 30¢; Springfield, Mass., 43¢; Winnipeg, Manitoba, Canada, 15.91¢; Richmond-Sunset, 31¢; and Annapolis, 33.6¢. As the various charges included in such accounting are dependent on numerous factors of local significance, such a tabulation is of little value except to compare, roughly, the total treatment costs in those plants employing digestion and elutriation with the Minneapolis-St. Paul plant. To arrive at better comparisons of items essential to obtaining economic filter operation further breaking down of such data is necessary.

The same statement is valid for Table 19(*b*), which is the first step in the process of breaking down sludge disposal costs. It is evident from Table 19(*a*) that the unit cost of chemicals and power varies materially at the different plants. Aside from these facts the sludge lifts differ in the various plants. At Minneapolis-St. Paul all filtration costs include the minor cost of sludge concentration and all costs from and after the point where sludge is pumped from the settling tanks to the point where sludge cake is discharged in the furnaces. The same principle is followed in the remainder of the installations listed in Table 19(*b*). This procedure necessarily includes the entire operating costs for elutriation in the installations using this process.

At Springfield the power is estimated roughly and is taken amply high. The total treatment cost already listed includes an item for depreciation and fixed charges beyond actual operating and maintenance costs. Although Table 19(*b*) has been adjusted to this fact, the higher power estimate has not been changed. This installation has higher sludge lifts than the other plants listed and includes the cost of all changes made during 1941, which was the first full year's operation of this relatively new plant. According to a private communication from J. D. McDonald, engineer in charge of this plant, the 1941 filtration costs included cost of extra ferric chloride and filter cloths purchased beyond 1941 needs and making various minor changes about the filters, conditioning mixer, and incinerator. Aside from these facts, running any vacuum filter to suit an incinerator's sensitive nature necessarily prolongs hours of filter operation and increases power. This was particularly true during the first year's operation at this plant.

At the other elutriation plants the cost of disposing of sludge cake by freight-car shipment or trucking is included. At Annapolis the sale of sludge cake paid for all trucking charges.

At Hartford, the filtration costs were 1¢ per capita higher in 1941 than in 1940 due to the material reduction in population served during 1941—150,000 against 201,400 in 1940. This decrease was brought about by the necessity of diverting a portion of the sewage flow from plant treatment while work on the Park River conduit was in progress.

It is to be regretted that the total cost of sludge filtration and incineration at Buffalo could not be broken down into the items listed in Tables 19(*b*) and 19(*d*), although other tabulated essential information is available.

As chemical and power costs for sludge filtration make up the major portion of total annual filter operating costs and the unit prices for chemicals and power vary materially in the plants listed, Table 19(*b*) should be tabulated with these costs uniform in all cases. However, it is simpler to present these essential items as in Tables 19(*c*) and 19(*d*).

In Table 19(*c*), Item No. 29 shows the per capita cost for chemicals placed on the price paid for ferric chloride at Minneapolis-St. Paul. As the Buffalo plant pays less for ferric chloride than does any other plant listed, the other per capita chemical costs are adjusted to the Buffalo price in Item No. 30. The differences in favor of the elutriation installations are striking. During 1942 the ferric consumption and costs at both Hartford and Springfield improved. At Hartford the improvement was principally due to securing and using more wash water in elutriation during the latter half of 1941. In fact, the filter yield was materially increased at the same time.

Table 19(*d*) is more or less self-explanatory. The total kilowatt-hours used for sludge filtration during 1941 are shown for all plants except Buffalo. In the elutriation installations the figures include all power used for elutriation. As the expression "population served per filter hour per day or year" has little meaning, due to the fact that different plants use different sizes of filter units, and measure filter hours in terms of each unit, it is rather obvious that filter hours per capita should be reduced to some identical filter area. For this reason, the annual filter hours per thousand population per 100 sq ft of area in the various plants is shown in Table 19(*d*). For example, Minneapolis-St. Paul, Baltimore, Buffalo, and the District of Columbia all use some multiple of 500-sq-ft filter units (see Table 19(*f*)), and the other plants listed use other sizes. Minneapolis-St. Paul recorded 39,919.1 filter hours in 1941 based on one filter unit of 500 sq ft in area serving 805,000 population. Hence, the population served by 100 sq ft of area during this time was $\frac{805{,}000}{5}$, or 161,000; and, for each 1,000 population per 100 sq ft, the filter hours were $\frac{39{,}919.1}{161} = 248$. The other plant data are calculated and listed in the same manner.

Dewatering less sludge solids at higher filter yields in the elutriation installations causes the power and filter hours to be notably lower in such cases. The lowest figure is for the smallest installation at Annapolis. At this plant the flow of digested sludge to elutriation is by gravity, the sludge lift to the vacuum filter is very low, and one or two other mechanized devices for moving and concentrating sludge are absent. Therefore, the power consumption should be relatively low.

Table 19(*e*) is self-explanatory. Again all of the essential factors relating to life of filter dressings, absence of acid washing, etc., are markedly in favor of the elutriation installations. Baltimore is not listed although it shows the lowest filter-cloth consumption of all—1,200 hr of cloth life, or only 2.5 sq ft of cloth dressing per 1,000 population per year. To keep filter yields up and ferric chloride consumption down at Winnipeg, wool cloth dressings are changed more frequently. The wool blankets used at this plant are manufactured locally and cost only $21.00 each. Annapolis uses canton flannel for its small 65-sq-ft filter. Each filter dressing costs just $1.19, or only $6.95 for the entire year of 1941. As a wool blanket lasted practically a full year and cost about $27.00, it is evident why canton flannel is used. Changing cloths on such a small filter unit is an easy and inexpensive task.

Table 19(*f*) relates to population actually served per filter hour per square foot of filter area, per filter operating day, in each plant listed for 1941. Although this essential figure is again materially in favor of the elutriated sludge plants, such a tabulated comparison is not fair to these plants for the simple reason that the vacuum filters in these plants operate only a fraction of the time listed for raw sludge dewatering at Minneapolis-St. Paul. If all the plants were operating on the Minneapolis-St. Paul time basis, without changing the coagulant dose used in 1941, and at the same filter yield, Table 19(*g*) would result. In other words, the average population per square foot per filter hour for all the elutriation installations would be almost ten times greater than at Minneapolis-St. Paul. Of course, it might be argued legitimately that such practice in the installations using only one or two vacuum filters would not allow time for filter repairs and redressing. However, if a spare filter were provided as a standby in such cases, the results would still be at least seven times greater; and if all plants began with approximately the same amount of raw sludge solids per capita, the digestion-elutriation installations would appear about four times better in this particular respect, which the writer considers one of the basic defects in the economy of dewatering large quantities of raw sludge in vacuum filters. As a matter of fact, Mr. Schroepfer's own analytical conclusions show that for Minneapolis-St. Paul the economics of filtration require still more filter area. It should be evident, from the study of Tables 19(*f*) and 19(*g*), that the tendency in the plants employing sludge digestion and elutriation is in the opposite direction—doing the dewatering job with the least amount of vacuum filter equipment and at the least annual operating expense. After all, these items are of material importance to the taxpayer.

Mr. Schroepfer leans to the opinion that filter equipment at Minneapolis-St. Paul may be eliminated by reducing the sludge moisture to 85% or 90%

and incinerating the raw sludge directly by increasing the amount of incineration equipment. Sludge digestion is essentially a wet combustion process itself in which the solids are mineralized before further dewatering. In view of the fact that the bacteria in this process do this work for nothing and with much less wear and tear on the relatively long-lived equipment involved, produce much less solids to be dewatered, and plenty of gaseous fuel for plant heating, the writer seriously questions the economics of raw sludge incineration compared to sludge digestion, elutriation, and vacuum filtration, especially on the sludges listed in Table 19(*a*). In fact, by using the latter procedure at Minneapolis-St. Paul, it is doubtful if final sludge incineration would be needed because the digested sludge cake would have a soil value that is out of the question with raw sludge.

According to the writer's estimates, destroying only 60% of the volatile matter in the unthickened raw sludge by digestion at Minneapolis-St. Paul, followed by elutriation of this sludge with river water and filtration, would save over $25,000 more a year in chemicals and power than the 1941 records show. With Mr. Schroepfer's natural genius for pushing filtration economies to the limit in practice, this estimate would undoubtedly be improved. In this estimate no account is taken of the savings due to elimination of lime incrustation, acid washing of filters, extra labor involved in these unnecessary items, frequent cloth changing, and the ultimate use of digestion gas for plant heating.

In the discussion of the relative economies of the vacuum filtration of raw and digested sludges, the argument that initial costs and fixed charges for digestion and elutriation space and equipment may offset the gains due to sludge mineralization has lost is validity in the numerous installations practicing this procedure. Mr. Fuhrman's paper on sludge dewatering at the District of Columbia clearly demonstrates this fact. Extensive filtration and incineration equipment for raw sludge is very short lived compared to digestion and elutriation tanks. Consequently, amortization charges against the former are much higher than against the latter. Notwithstanding these facts the argument has been further invalidated through the recent development of the relatively inexpensive "coilless" digester by A. M. Rawn, M. Am. Soc. C. E., whose contribution (28) to the advance of sludge digestion is outstanding. Expensive construction of tanks with floating covers, hot water coils, and elaborate heat exchanging equipment may be eliminated. As for tank space required for countercurrent or multi-stage elutriation, this is relatively small (0.02 to 0.1 of that installed for digestion). According to the percentage of solids present in the digested sludge, water ratio used for solids washing, and operating hours per operating day, it varies from 0.02 to 0.2 cu ft per capita, the latter value being for thin digested waste activated sludge and eight operating hours per filter operating day.

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Bibliography.—

(27) *Proceedings*, Am. Soc. C. E., November, 1942, p. 1570.
(28) *Waterworks and Sewerage*, July, 1942, p. 310.